

Week 5: Advanced Encryption Standard

Click <http://www.nist.gov/aes>

The screenshot shows a web browser window with the address bar displaying "NIST http://csrc.nist.gov/archive/aes/index.html". The browser's search bar contains "Google". Below the browser window, a yellow banner contains the text: "Archive info: This page is provided for historical purposes. It is no longer maintained and may contain errors or inaccuracies." The main content area shows the NIST Cryptographic Toolkit website. The page title is "AES" (Advanced Encryption Standard). The main heading is "FIPS". The text below reads: "NIST is pleased to announce the approval of the Federal Information Processing Standard (FIPS) for the Advanced Encryption Standard, FIPS-197. This standard specifies Rijndael as a FIPS-approved symmetric encryption algorithm that may be used by U.S. Government organizations (and others) to protect sensitive information. Federal agencies should also see OMB guidance." Below this text is a list of links: "Federal Register Announcement" of the FIPS, and "FIPS 197 [PS][PDF]". The section "Additional AES information" includes links for "Rijndael (AES algorithm) Information", "NIST's Press Release and Report on the AES Selection", and "AES OIDs and ASN.1 module". A left sidebar contains a search bar and various navigation links such as "Home", "Library", "Services", "Events", "Advisories", "Contact", "Site Map", "AES", "FIPS", "AES Code & Vectors", "AES Press Release", "NIST's AES Report", "Archived AES Pages", "Modes of Operation", "Cryptographic Toolkit", "Project Sites", "CMVP", "PKI", "Common Criteria", "NIAP", "Security Practices", and "Publications".

History of AES

□ Calendar

- 1997 : Call For AES Candidate Algorithms by NIST
 - 128-bit Block cipher
 - 128/192/256-bit keys
 - Worldwide-royalty free
 - **More secure than Triple DES**
 - **More efficient than Triple DES**
- 1998 : 1st Round Candidates – 15 Algorithms
 - Mars, Twofish, RC6, SAFER+, HPC, CAST256, DEAL, Frog, Magenta, Rijndael, DFC, Serpent, Crypton, E2, LOKI97
- 1999 : 2nd Round Candidates – 5 Algorithms
 - **MARS, RC6, Rijndael, Serpent, and Twofish**
- 2000. 10 : **Rijndael** selected as the finalist
- 2001. 12: official publication as FIPS PUB 197

* NIST: National Institute of Standards and Technology

AES Contest – 1st Round Candidates

Cipher	Submitted by	Country
CAST-256	Entrust	Canada
Crypton	Future Systems	Korea [†]
Deal	Outerbridge	Canada [†]
DFC	ENS–CNRS	France
E2	NTT	Japan
Frog*	TecApro	Costa Rica
HPC*	Schroepfel	USA
LOKI97*	Brown, Pieprzyk, Seberry	Australia
Magenta	Deutsche Telekom	Germany
Mars	IBM	USA [†]
RC6	RSA	USA [†]
Rijndael*	Daemen, Rijmen	Belgium [‡]
Safer+*	Cylink	USA [†]
Serpent*	Anderson, Biham, Knudsen	UK, Israel, Norway
Twofish*	Counterpane	USA [†]

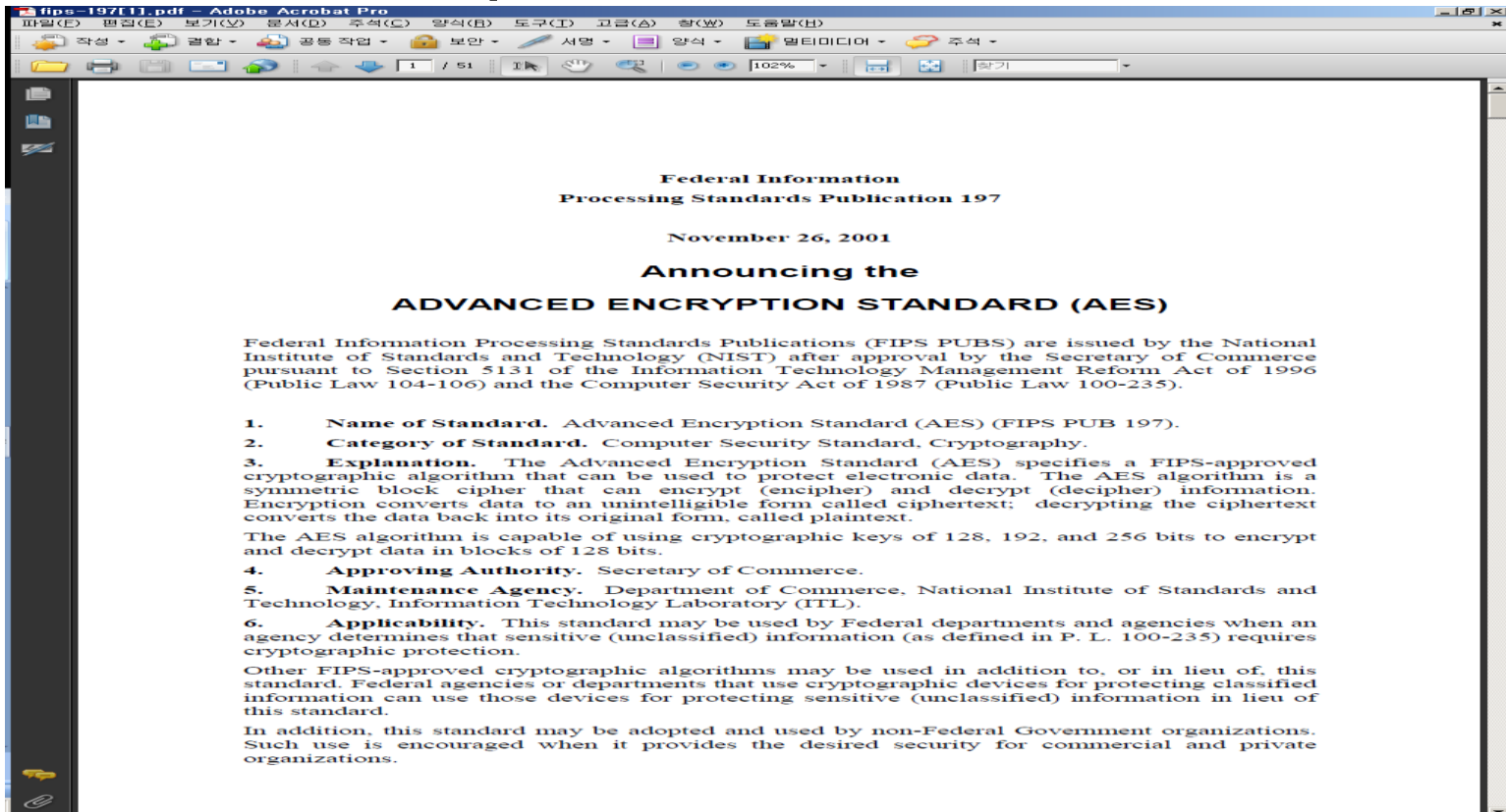
* Placed in the public domain; † and foreign designers; ‡ foreign influence

AES Contest – 2nd Round Candidates

Cipher	Submitter	Structure	Nonlinear Component
MARS	IBM	Feistel structure	Sbox DD-Rotation
RC6	RSA Lab.	Feistel structure	Rotation
Rijndael	Daemen, Rijmen	SPN structure	Sbox
Serpent	Anderson, Biham, Knudsen	SPN structure	Sbox
Twofish	Schneier et. al	Feistel structure	Sbox

AES Contest – Finalist (1/2)

- 2000. 10 : [Rijndael](#) selected as the finalist
- 2001. 12: official publication as FIPS PUB 197



Joan Daemen and Vincent Rijmen, "[The Design of Rijndael, AES – The Advanced Encryption Standard](#)", Springer, 2002, ISBN 3-540-42580-2



AES Contest – Finalist (2/2)

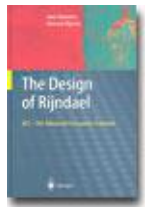
The Advanced Encryption Standard (AES) is a National Institute of Standards and Technology specification for the encryption of electronic data.

How secure is AES? The **general consensus is that it is the most secure encryption algorithm available**. AES has been subjected to more scrutiny than any other encryption algorithm to date. On both a theoretical and practical basis, AES is considered "secure" in the sense that the only effective way to crack it is through a brute-force generation of all possible keys. With a key size of 256 bits, no known brute-force attack can break AES in a reasonable amount of time (it would take years even on the fastest systems available).

In cryptography, the Advanced Encryption Standard (AES) is an encryption standard adopted by the U.S. government. The AES ciphers have been analyzed extensively and are now used worldwide.

AES was announced by National Institute of Standards and Technology (NIST) as U.S. FIPS PUB 197 (FIPS 197) on November 26, 2001 after a 5-year standardization process in which fifteen competing designs were presented and evaluated before Rijndael was selected as the most suitable (see Advanced Encryption Standard process for more details). It became effective as a Federal government standard on May 26, 2002 after approval by the Secretary of Commerce. It is available in many different encryption packages. AES is the first publicly accessible and open cipher **approved by the NSA for top secret information**.

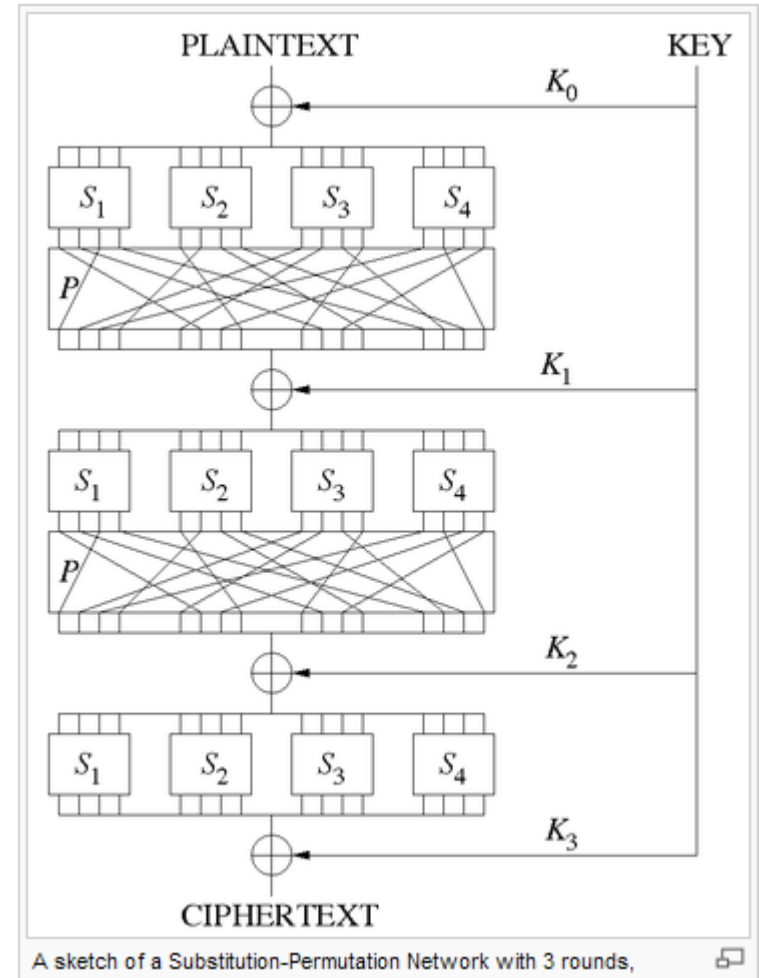
Security Goals of Rijndael



- **K-secure**
 - No shortcut attacks key-recover attack **faster than key-exhaustive search**
 - No symmetry property such as **complementary** in DES
 - **No non-negligible classes of weak key** as in IDEA
 - **No Related-key attacks**
- **Hermetic**
 - No weakness found for the majority of block ciphers with same block and key length
- ***Rijndael is k-secure and hermetic***

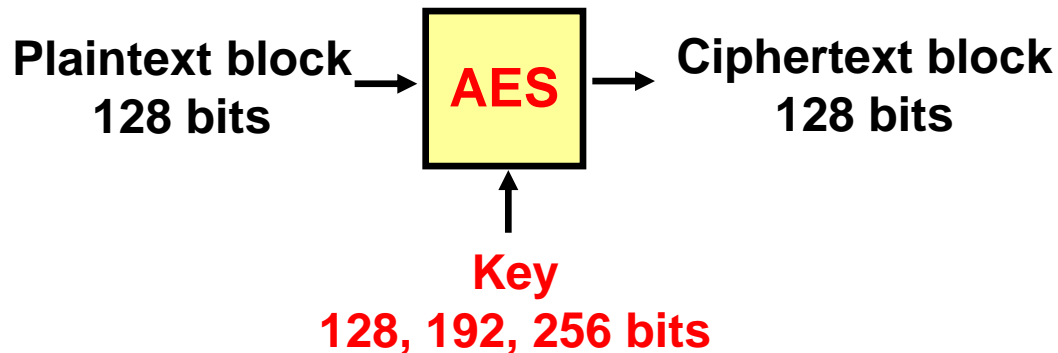
SPN- Structure Block Cipher

- **More constraints on the round function: must be invertible**
 - ✓ (Invertible) Nonlinear Layer: Confusion
 - ✓ (Invertible) Linear Layer: Diffusion
- **Relatively new architecture and faster than Feistel-structure**
- **Parallel computation**
- **Typically $E \neq D$**



Advanced Encryption Standard (AES)

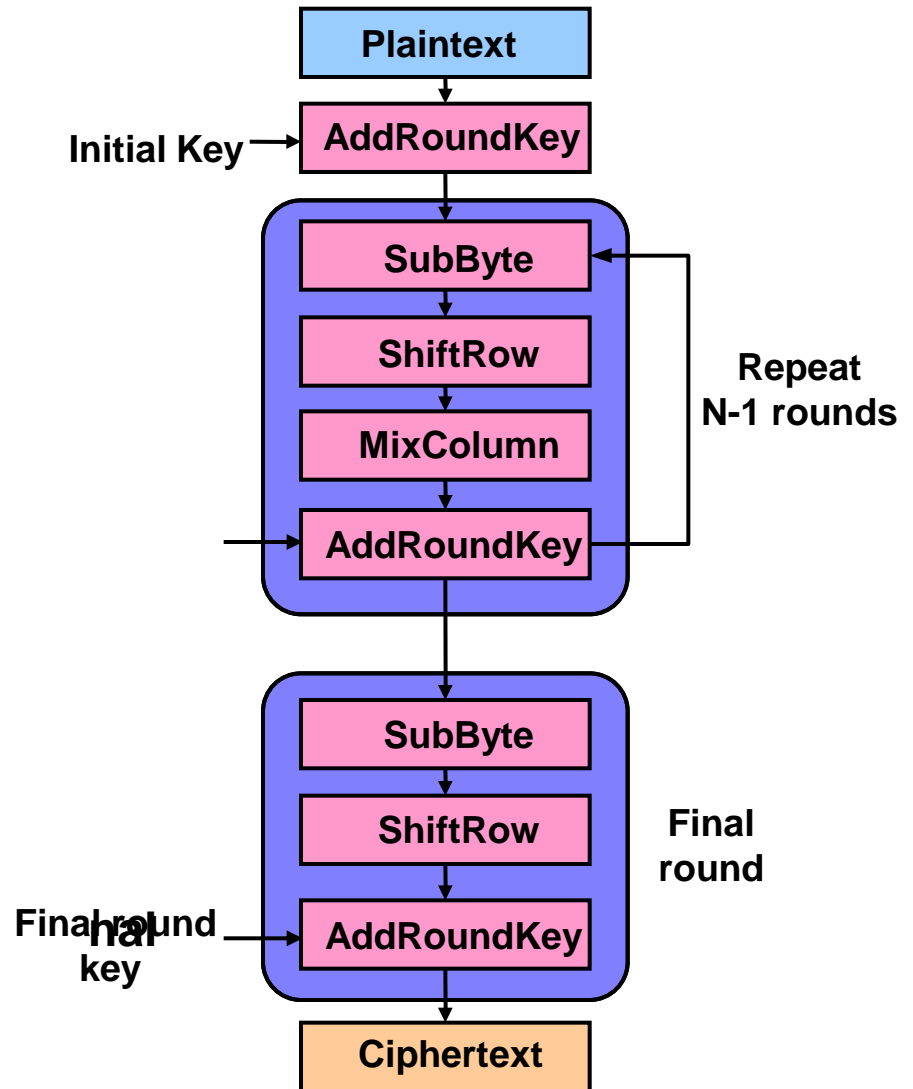
AES Parameters



H/W : Get AES program (AES-128, 192 or 256) and check the Encryption and Decryption by choosing a plaintext and key at random. Submit your source code and test vector in hardcopy.

AES Architecture

- **SPN-type block cipher**
- **Block size = 128 bits**
- **Key size / No. round**
 - 128 bits → 10 rounds
 - 192 bits → 12 rounds
 - 256 bits → 14 rounds
- **Round transformation**
 - **SubBytes**
 - **ShiftRow**
 - **MixColumn**
 - **AddRoundKey**



Finite Field

- **Every nonzero element has multiplicative inverse.**
- **Prime number, Z_p**
 - **gcd, Extended Euclidean Algorithm**
- **Representation of n-bit data**
 - **Binary**
 - **Hexadecimal**
 - **Polynomial**
- **$GF(p)$, $GF(2^n)$ over irreducible polynomial**
 - **Ex) $GF(2^3) / x^3 + x + 1$**
 - **Addition and multiplication**

 - **AES uses $GF(2^8) / f(x) = x^8 + x^4 + x^3 + x + 1$**

Finite Field - Addition

The addition of two elements in a finite field is achieved by “adding” the coefficients for the corresponding powers in the polynomials for the two elements. The addition is performed with the XOR operation (denoted by \oplus) - i.e., modulo 2 - so that $1 \oplus 1 = 0$, $1 \oplus 0 = 1$, and $0 \oplus 0 = 0$. Consequently, subtraction of polynomials is identical to addition of polynomials.

Alternatively, addition of finite field elements can be described as the modulo 2 addition of corresponding bits in the byte. For two bytes $\{a_7a_6a_5a_4a_3a_2a_1a_0\}$ and $\{b_7b_6b_5b_4b_3b_2b_1b_0\}$, the sum is $\{c_7c_6c_5c_4c_3c_2c_1c_0\}$, where each $c_i = a_i \oplus b_i$ (i.e., $c_7 = a_7 \oplus b_7$, $c_6 = a_6 \oplus b_6$, ... $c_0 = a_0 \oplus b_0$).

For example, the following expressions are equivalent to one another:

$$(x^6 + x^4 + x^2 + x + 1) + (x^7 + x + 1) = x^7 + x^6 + x^4 + x^2 \quad (\text{polynomial notation});$$

$$\{01010111\} \oplus \{10000011\} = \{11010100\} \quad (\text{binary notation});$$

$$\{57\} \oplus \{83\} = \{d4\} \quad (\text{hexadecimal notation}).$$

Finite Field - Multiplication

In the polynomial representation, multiplication in $GF(2^8)$ (denoted by \bullet) corresponds with the multiplication of polynomials modulo an irreducible polynomial of degree 8. A polynomial is irreducible if its only divisors are one and itself. For the AES algorithm, this irreducible polynomial is

$$m(x) = x^8 + x^4 + x^3 + x + 1, \quad (4.1)$$

or $\{01\} \{1b\}$ in hexadecimal notation.

For example, $\{57\} \bullet \{83\} = \{c1\}$, because

$$\begin{aligned} (x^6 + x^4 + x^2 + x + 1)(x^7 + x + 1) &= x^{13} + x^{11} + x^9 + x^8 + x^7 + \\ &\quad x^7 + x^5 + x^3 + x^2 + x + \\ &\quad x^6 + x^4 + x^2 + x + 1 \\ &= x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \end{aligned}$$

and

$$\begin{aligned} x^{13} + x^{11} + x^9 + x^8 + x^6 + x^5 + x^4 + x^3 + 1 \text{ modulo } (x^8 + x^4 + x^3 + x + 1) \\ = x^7 + x^6 + 1. \end{aligned}$$

The modular reduction by $m(x)$ ensures that the result will be a binary polynomial of degree less than 8, and thus can be represented by a byte. Unlike addition, there is no simple operation at the byte level that corresponds to this multiplication.

The multiplication defined above is associative, and the element $\{01\}$ is the multiplicative identity. For any non-zero binary polynomial $b(x)$ of degree less than 8, the multiplicative inverse of $b(x)$, denoted $b^{-1}(x)$, can be found as follows: the extended Euclidean algorithm [7] is used to compute polynomials $a(x)$ and $c(x)$ such that

$$b(x)a(x) + m(x)c(x) = 1. \quad (4.2)$$

Hence, $a(x) \bullet b(x) \text{ mod } m(x) = 1$, which means

$$b^{-1}(x) = a(x) \text{ mod } m(x). \quad (4.3)$$

Moreover, for any $a(x)$, $b(x)$ and $c(x)$ in the field, it holds that

$$a(x) \bullet (b(x) + c(x)) = a(x) \bullet b(x) + a(x) \bullet c(x).$$

It follows that the set of 256 possible byte values, with XOR used as addition and the multiplication defined as above, has the structure of the finite field $GF(2^8)$.

Finite Field – Multiplication by x

Multiplying the binary polynomial defined in equation (3.1) with the polynomial x results in

$$b_7x^8 + b_6x^7 + b_5x^6 + b_4x^5 + b_3x^4 + b_2x^3 + b_1x^2 + b_0x. \quad (4.4)$$

The result $x \cdot b(x)$ is obtained by reducing the above result modulo $m(x)$, as defined in equation (4.1). If $b_7 = 0$, the result is already in reduced form. If $b_7 = 1$, the reduction is accomplished by subtracting (i.e., XORing) the polynomial $m(x)$. It follows that multiplication by x (i.e., $\{00000010\}$ or $\{02\}$) can be implemented at the byte level as a left shift and a subsequent conditional bitwise XOR with $\{1b\}$. This operation on bytes is denoted by `xtime()`. Multiplication by higher powers of x can be implemented by repeated application of `xtime()`. By adding intermediate results, multiplication by any constant can be implemented.

For example, $\{57\} \cdot \{13\} = \{fe\}$ because

$$\begin{aligned} \{57\} \cdot \{02\} &= \text{xtime}(\{57\}) = \{ae\} \\ \{57\} \cdot \{04\} &= \text{xtime}(\{ae\}) = \{47\} \\ \{57\} \cdot \{08\} &= \text{xtime}(\{47\}) = \{8e\} \\ \{57\} \cdot \{10\} &= \text{xtime}(\{8e\}) = \{07\}, \end{aligned}$$

thus,

$$\begin{aligned} \{57\} \cdot \{13\} &= \{57\} \cdot (\{01\} \oplus \{02\} \oplus \{10\}) \\ &= \{57\} \oplus \{ae\} \oplus \{07\} \\ &= \{fe\}. \end{aligned}$$

Round Transformation – S-box

The `SubBytes()` transformation is a non-linear byte substitution that operates independently on each byte of the State using a substitution table (S-box). This S-box (Fig. 7), which is invertible, is constructed by composing two transformations:

1. Take the multiplicative inverse in the finite field $GF(2^8)$, described in Sec. 4.2; the element $\{00\}$ is mapped to itself.
2. Apply the following affine transformation (over $GF(2)$):

$$b'_i = b_i \oplus b_{(i+4)\text{mod}8} \oplus b_{(i+5)\text{mod}8} \oplus b_{(i+6)\text{mod}8} \oplus b_{(i+7)\text{mod}8} \oplus c_i \quad (5.1)$$

for $0 \leq i < 8$, where b_i is the i^{th} bit of the byte, and c_i is the i^{th} bit of a byte c with the value $\{63\}$ or $\{01100011\}$. Here and elsewhere, a prime on a variable (e.g., b'_i) indicates that the variable is to be updated with the value on the right.

In matrix form, the affine transformation element of the S-box can be expressed as:

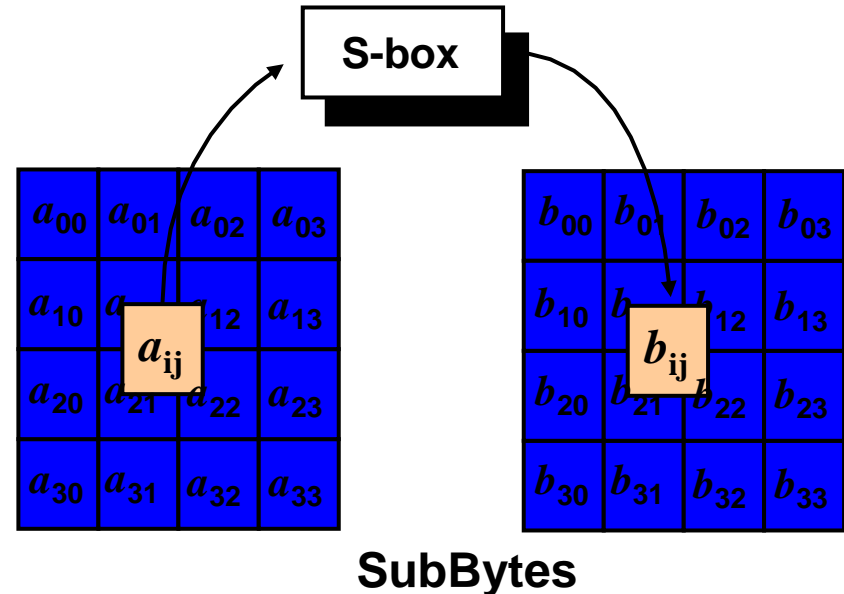
$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

The S-box used in the `SubBytes()` transformation is presented in hexadecimal form in Fig. 7.

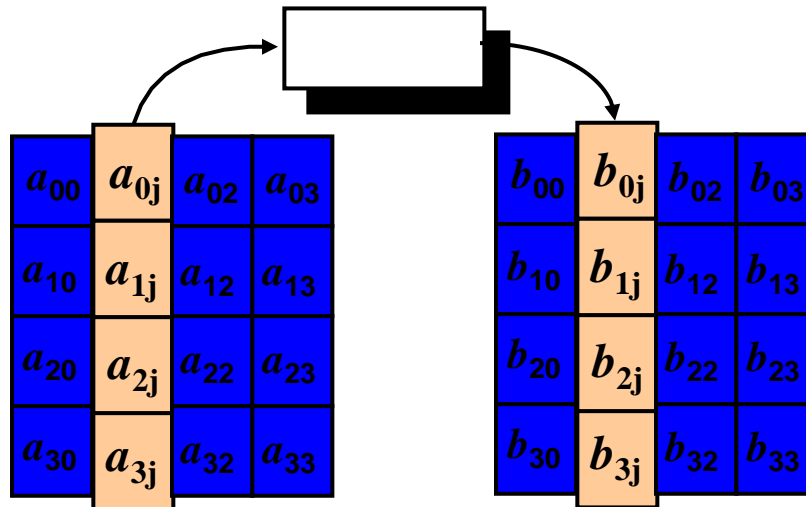
For example, if $s_{11} = \{53\}$, then the substitution value would be determined by the intersection of the row with index '5' and the column with index '3' in Fig. 7. This would result in s'_{11} having a value of $\{e4\}$.

		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	bl	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Figure 7. S-box: substitution values for the byte xy (in hexadecimal format).



Round Transformation – Mixcolumns



The `MixColumns()` transformation operates on the State column-by-column, treating each column as a four-term polynomial as described in Sec. 4.3. The columns are considered as polynomials over $\text{GF}(2^8)$ and multiplied modulo $x^4 + 1$ with a fixed polynomial $a(x)$, given by

$$a(x) = \{03\}x^3 + \{01\}x^2 + \{01\}x + \{02\}. \quad (5.5)$$

As described in Sec. 4.3, this can be written as a matrix multiplication. Let

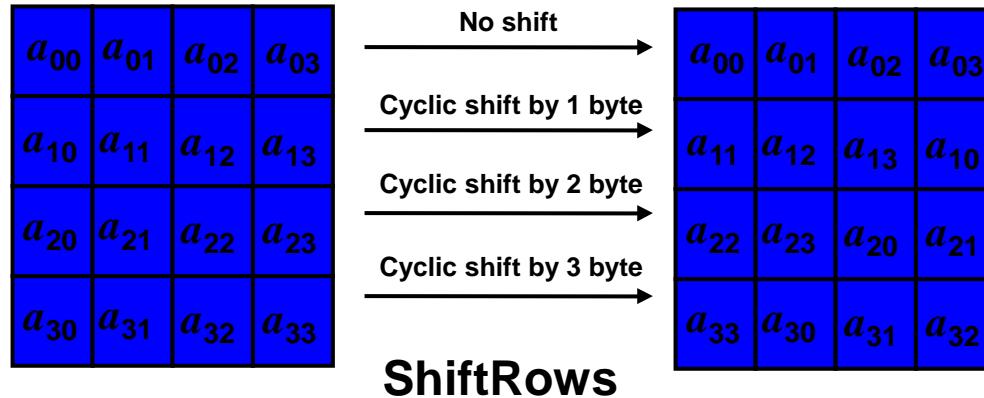
$$s'(x) = a(x) \otimes s(x):$$

$$\begin{bmatrix} s'_{0,c} \\ s'_{1,c} \\ s'_{2,c} \\ s'_{3,c} \end{bmatrix} = \begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,c} \\ s_{1,c} \\ s_{2,c} \\ s_{3,c} \end{bmatrix} \quad \text{for } 0 \leq c < Nb. \quad (5.6)$$

As a result of this multiplication, the four bytes in a column are replaced by the following:

$$\begin{aligned} s'_{0,c} &= (\{02\} \cdot s_{0,c}) \oplus (\{03\} \cdot s_{1,c}) \oplus s_{2,c} \oplus s_{3,c} \\ s'_{1,c} &= s_{0,c} \oplus (\{02\} \cdot s_{1,c}) \oplus (\{03\} \cdot s_{2,c}) \oplus s_{3,c} \\ s'_{2,c} &= s_{0,c} \oplus s_{1,c} \oplus (\{02\} \cdot s_{2,c}) \oplus (\{03\} \cdot s_{3,c}) \\ s'_{3,c} &= (\{03\} \cdot s_{0,c}) \oplus s_{1,c} \oplus s_{2,c} \oplus (\{02\} \cdot s_{3,c}). \end{aligned}$$

Round Transformation – ShiftRows



In the `ShiftRows()` transformation, the bytes in the last three rows of the State are cyclically shifted over different numbers of bytes (offsets). The first row, $r = 0$, is not shifted.

Specifically, the `ShiftRows()` transformation proceeds as follows:

$$s'_{r,c} = s_{r,(c+shift(r,Nb)) \bmod Nb} \quad \text{for } 0 < r < 4 \quad \text{and} \quad 0 \leq c < Nb, \quad (5.3)$$

where the shift value $shift(r,Nb)$ depends on the row number, r , as follows (recall that $Nb = 4$):

$$shift(1,4) = 1; \quad shift(2,4) = 2; \quad shift(3,4) = 3. \quad (5.4)$$

This has the effect of moving bytes to “lower” positions in the row (i.e., lower values of c in a given row), while the “lowest” bytes wrap around into the “top” of the row (i.e., higher values of c in a given row).

AddRoundKey

The input block is XOR-ed with the round key

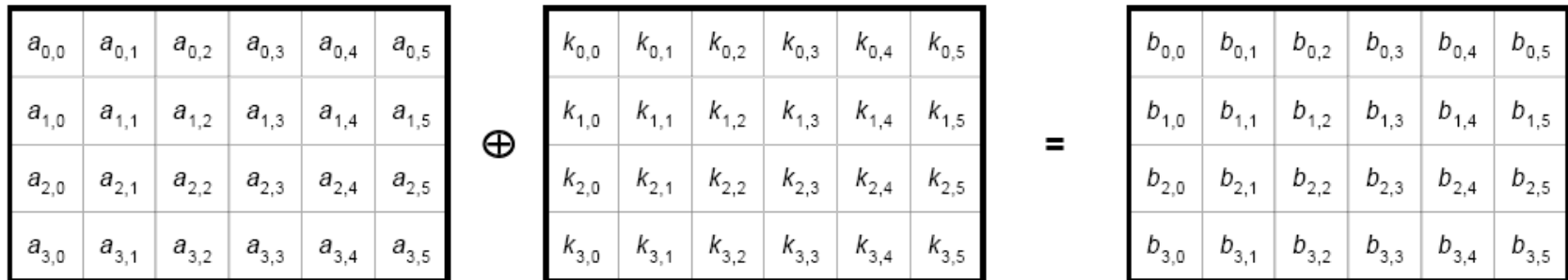


Figure 5: In the key addition the Round Key is bitwise EXORed to the State.

Best Known Attack

- [Related-key attack](#) can break 256-bit AES with a complexity of 2^{119} , which is faster than brute force but is still infeasible. 192-bit AES can also be defeated in a similar manner, but at a complexity of 2^{176} . 128-bit AES is not affected by this attack.
- [Chosen-plaintext attack](#) can break 8 rounds of 192- and 256-bit AES, and 7 rounds of 128-bit AES, although the workload is impractical at $2^{128} \sim 2^{119}$. (Ferguson et al., 2000).

Feistel-Network vs. SP Network (Summary)

Feistel-Network

64-bit block

- DES/3DES
- BLOWFISH
- CAST128
- RC5

128-bit block

- SEED
- TWOFISH
- CAST256
- RC6
- MARS

- Fewer constraints on the round function
- More cryptanalytic experience
- Serial in nature
- Typically $E = D$ with round keys in reverse order

SP Network

64-bit block 128-bit block

- SAFER
- SAFER+
- IDEA

- AES
- CRYPTON
- SERPENT

- More constraints on the round function: must be invertible
- Less cryptanalytic experience: relatively new architecture
- more parallel computation
- Typically $E \neq D$