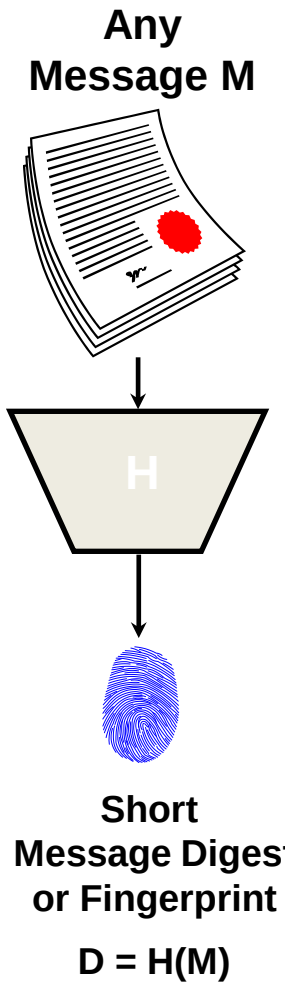
Week 12: Hash Functions and MAC

1. Introduction
Hash Functions vs. MAC

Hash Functions



❖ Hash Function

- ✓ Generate a fixed length "Fingerprint" for an arbitrary length message.
- ✓ No Key involved.
- ✓ Must be at least One-way to be useful.

❖ Applications

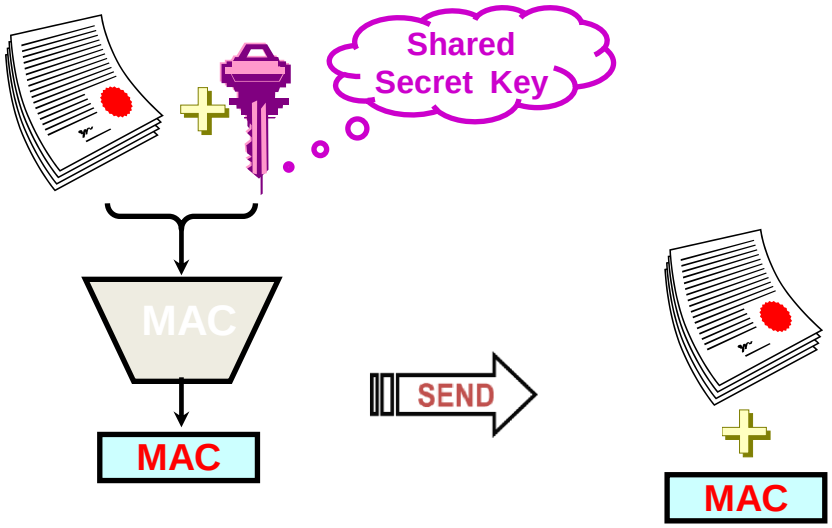
- ✓ Keyed hash: MAC/ICV generation.
- ✓ Unkeyed hash: digital signature, password file, key stream / pseudo-random number generator, etc.

❖ Constructions

- ✓ Iterated hash functions (MD4-family hash functions): MD5, SHA-1, SHA-2, HAVAL, HAS160, etc.
- ✓ Hash functions based on block ciphers: MDC(Manipulation Detection Code)

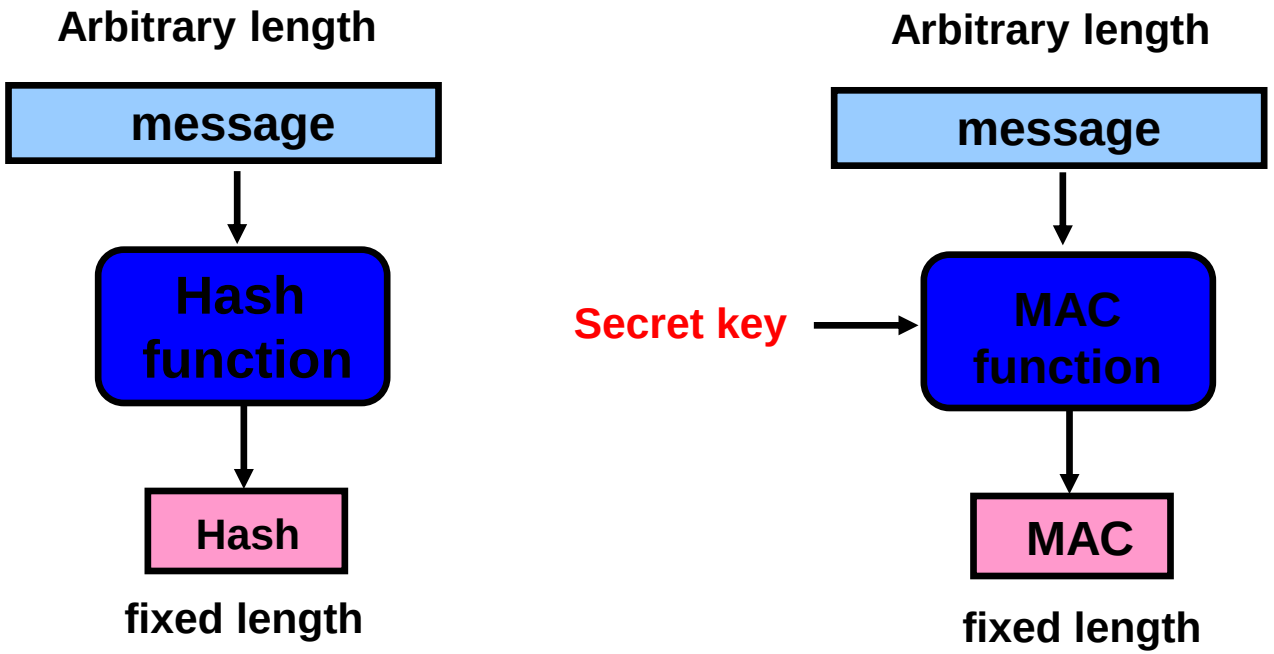
Message Authentication Code (MAC)

- **MAC**
  - ✓Generate a fixed length MAC for an arbitrary length message
  - ✓A keyed hash function
  - ✓Message origin authentication
  - ✓Message integrity
  - ✓Entity authentication



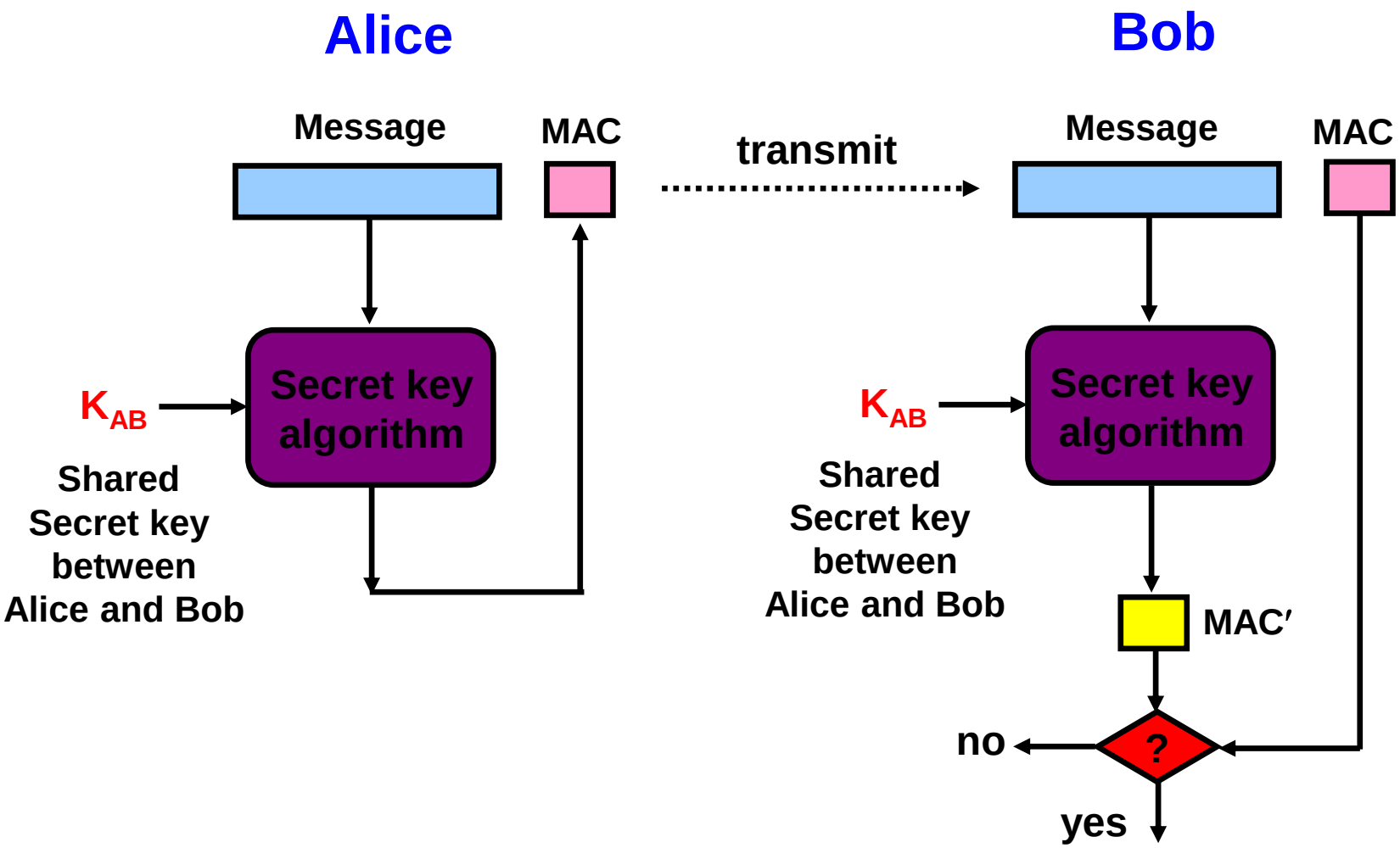
- **Constructions**
  - ✓Keyed hash: HMAC
  - ✓Block cipher: CBC-MAC

Comparison of Hash Function & MAC

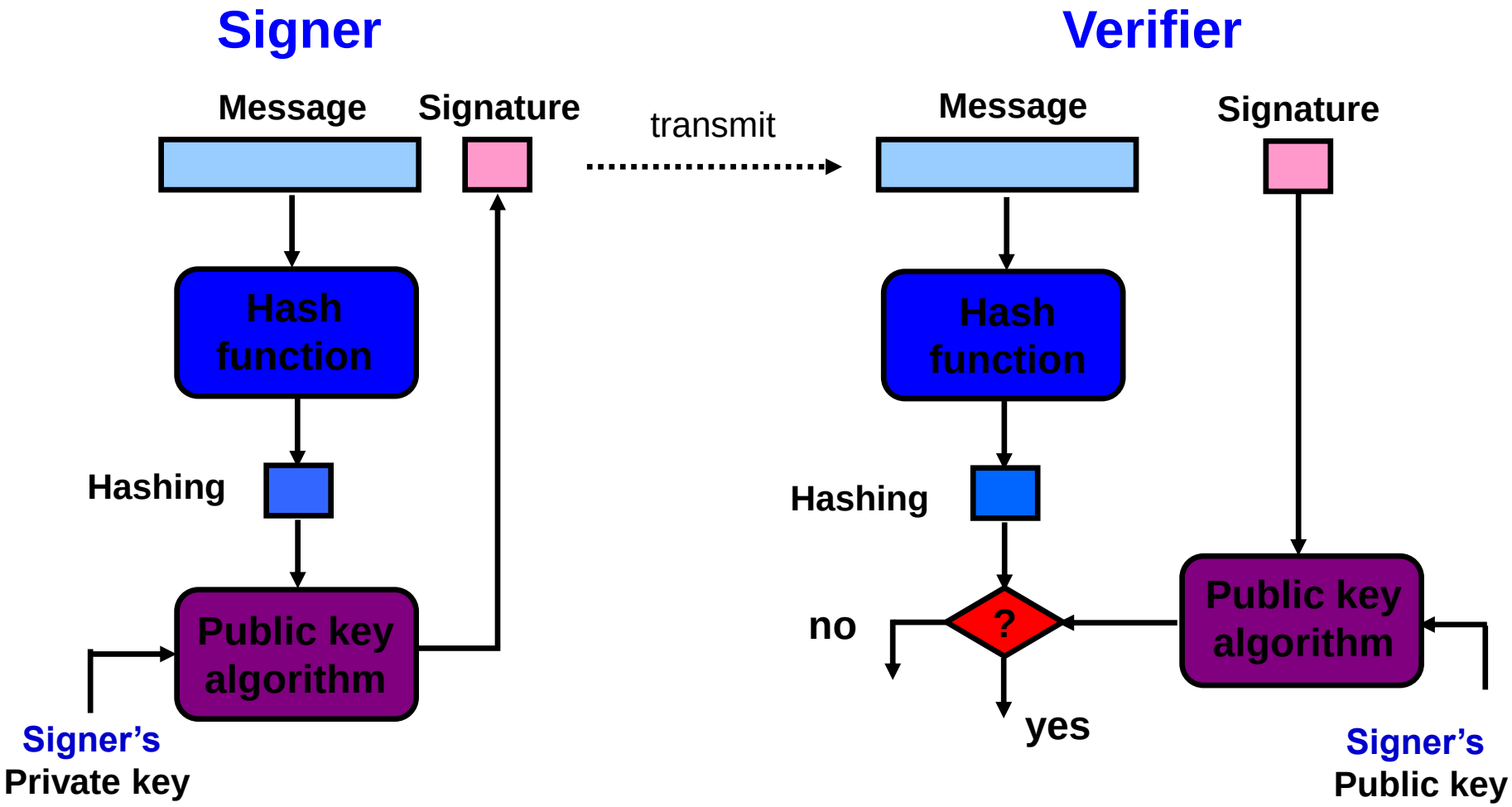


- Easy to compute
- Compression: arbitrary length input to fixed length output
- Unkeyed function vs. Keyed function

Message Authentication using MAC



Digital Signature with Hash Function



MAC and Digital Signature (Summary)

❖MAC (Message Authentication Code)

- Generated and verified by a secret key algorithm
- Message origin authentication & Message integrity
- Schemes
  - ✓ Keyed hash: HMAC
  - ✓ Block cipher: CBC-MAC,

❖Digital Signature

- Generated and verified by a public key algorithm and a hash function
- Message origin authentication & Message integrity
- Non-repudiation
- Schemes
  - ✓ Hash + Digital signature algorithm
  - ✓ RSA-PSS, DSA, etc.

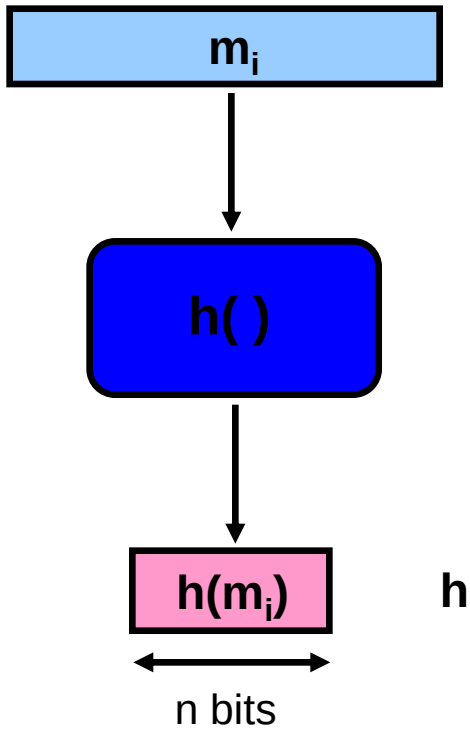
2. Hash Functions

Hash Functions – Requirements

- **Efficient Computation**
- **Security Properties**
  - **Preimage resistance** (One-wayness) :
    - Given *y*, it is computationally infeasible to find any input *x* such that $y = h(x)$
  - **2nd preimage resistance** (**Weak collision resistance**) :
    - Given *x*, it is computationally infeasible to find another input $x' \neq x$ such that $h(x) = h(x')$
  - **Collision resistance** (**Strong collision resistance**) :
    - It is computationally infeasible to find any two distinct inputs *x* and *x'* such that $h(x) = h(x')$

BFA on OW Hash Function (Pre-image Attack)

Given y,
find m such that
h(m) = y



for i = 1, 2, . . . 2^n

Arbitrary message, m_i
or
m_j of the same meaning ?

$h(m_i)$ = y ?

Multiple Messages with Same Meaning

I state / confirm thereby that I borrowed / received $10,000 / ten thousand dollars from

Mr. / Dr. Kris / Krzysztof Gaj on October 15, / 15 October 2001. This money / amount of money

should / is required to be returned / given back to Mr. / Dr. Gaj by November 30, / 30 November 2001.

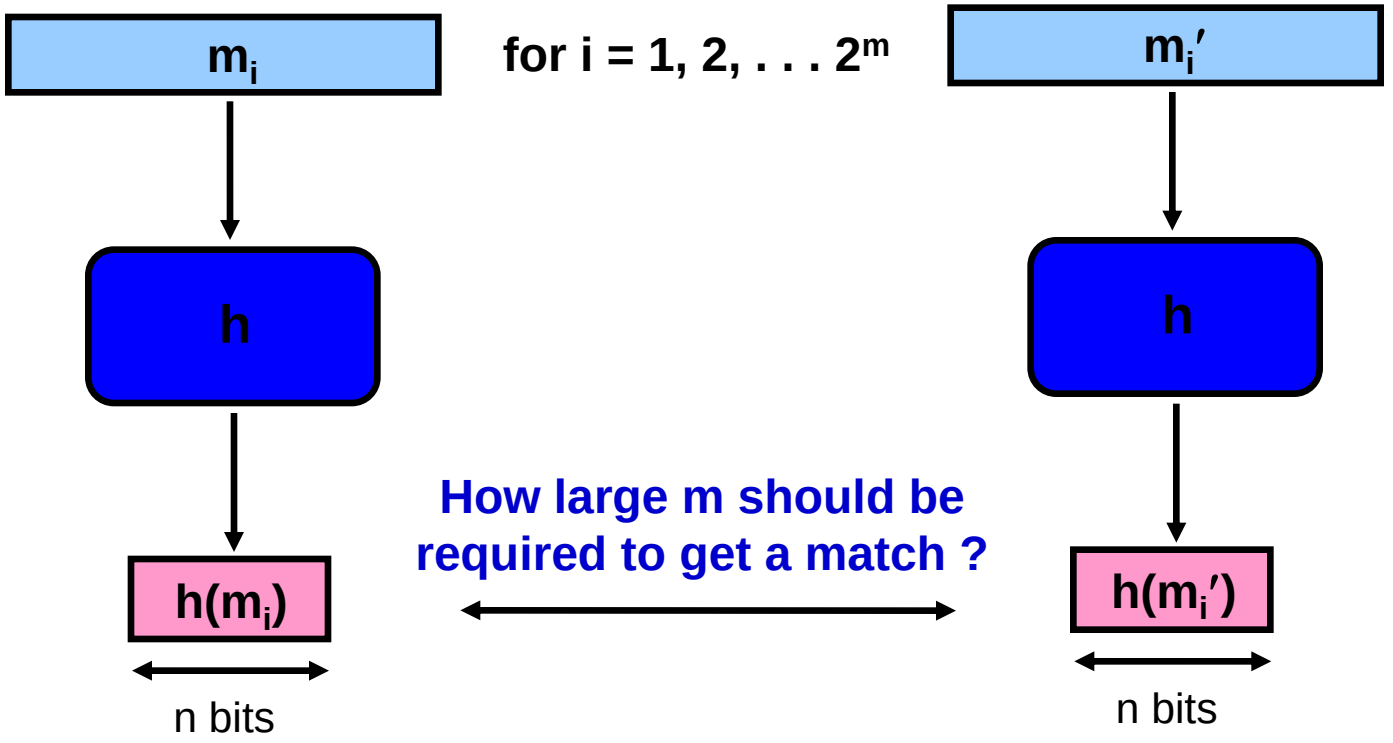
11 different positions of similar expressions

⇓

2^{11} different messages of the same meaning

Collision in Collision-Resistant Hash Function

Find any two distinct messages m, m′ such that h(m) = h(m′).



Birthday Paradox

How many students there must be in a class for there be a greater than 50% chance that

1. **One of the students shares the teacher's birthday ? (complexity breaking one-wayness)**

   $365/2 \approx 188$

2. **Any two of the students share the same birthday ? (complexity breaking collision resistance)**

   $1 - 365 \times 364 \times \ldots \times (365-k+1) / 365^k > 0.5 \Rightarrow k \approx 23$

In general, the probability of a match being found when k samples are randomly selected between 1 and n equals

$$1 - \frac{n!}{(n-k)!\,n^k} > 1 - e^{-\frac{k(k-1)}{2n}}$$

One Million $ Hardware Brute Force Attack

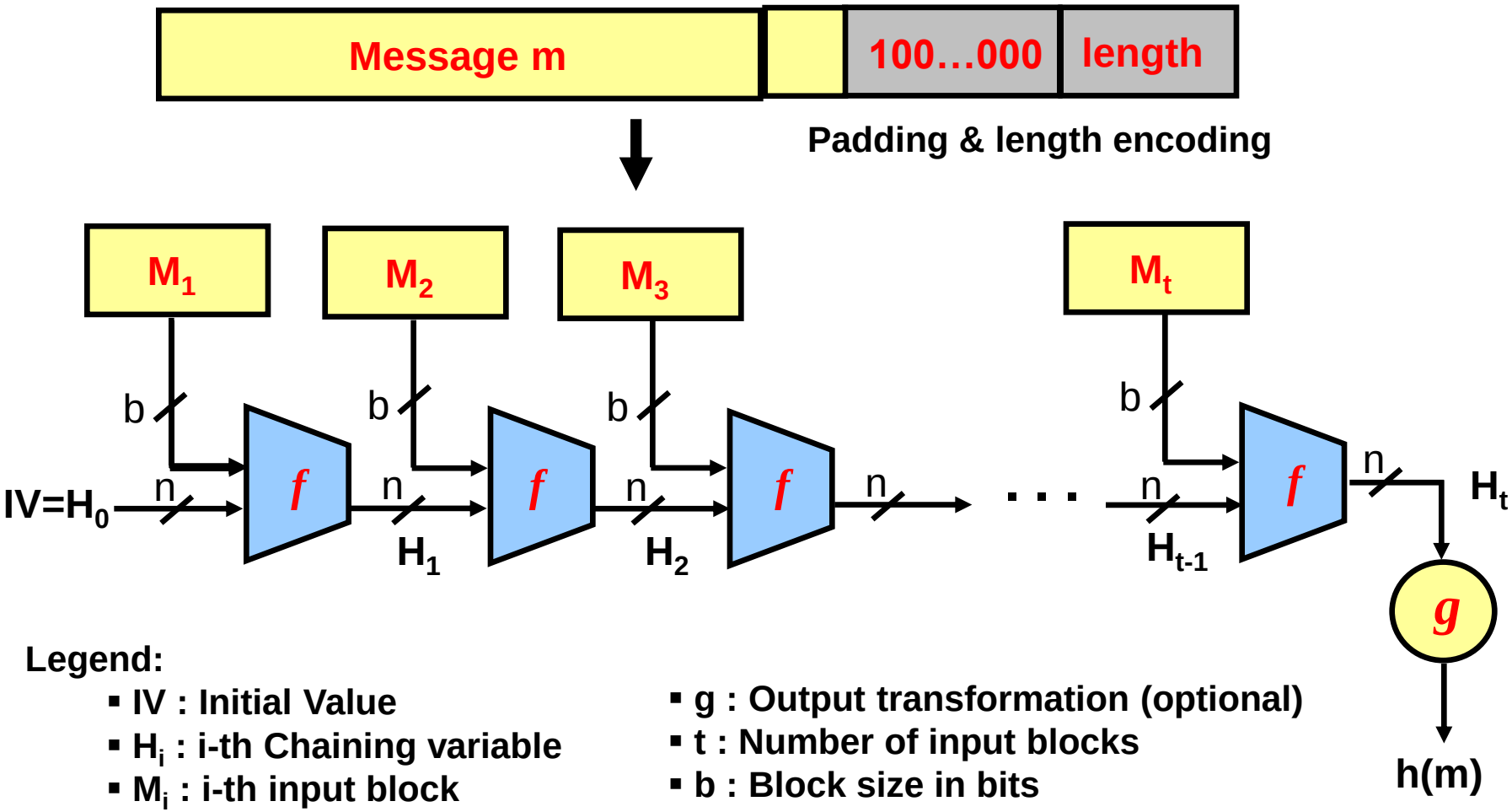
❖ **One-Way Hash Functions (complexity = 2^n)**

	n = 64	n = 80	n = 128
Year 2001	4 days	718 years	10^{17} years

❖ **Collision-Resistant Hash Functions (complexity = $2^{n/2}$)**

	n = 128	n = 160	n = 256
Year 2001	4 days	718 years	10^{17} years

Construction of Hash Function (1/2)

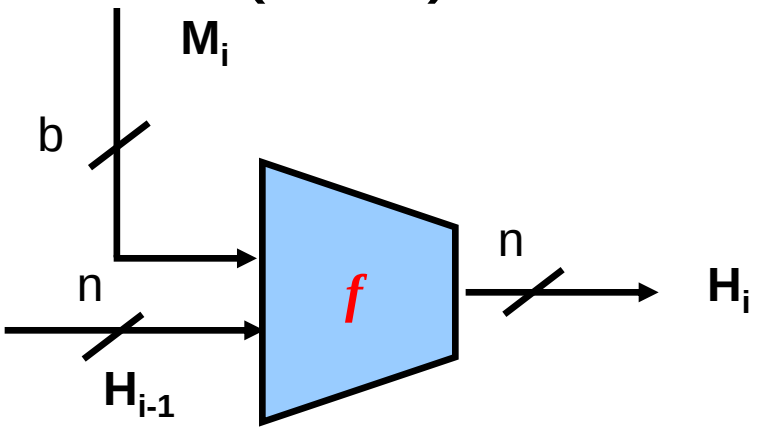


Legend:

- **IV : Initial Value**
- **H_i : i-th Chaining variable**
- **M_i : i-th input block**
- **f : Compression function**
- **g : Output transformation (optional)**
- **t : Number of input blocks**
- **b : Block size in bits**
- **n : Hash code size in bits**

Construction of Hash Function (2/2)

Compression Function
(fixed-size hash function)



Entire hash

$H_0 = IV$

$H_i = f(H_{i-1}, M_i)$ for $1 \le i \le t$

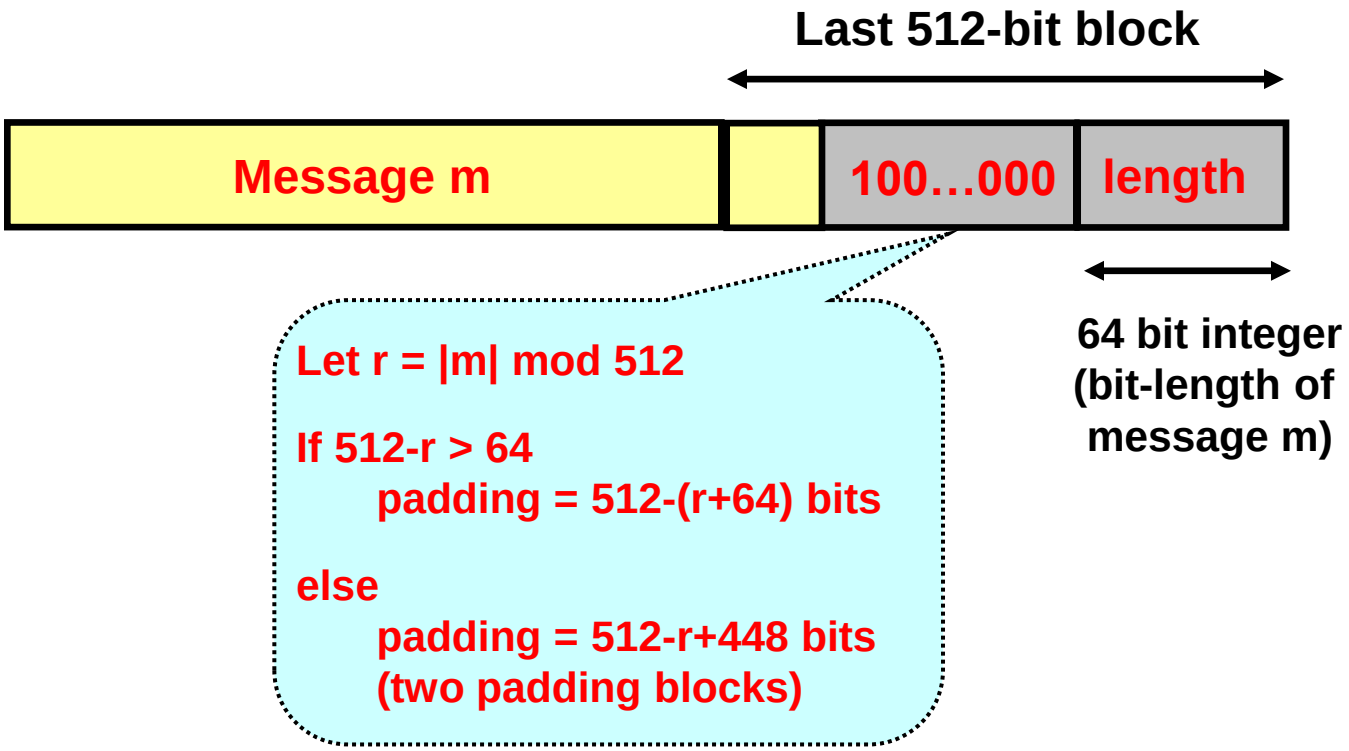
$H(m) = g(H_t)$

Fact(by Merkle-Damgård) MD-strengthening.

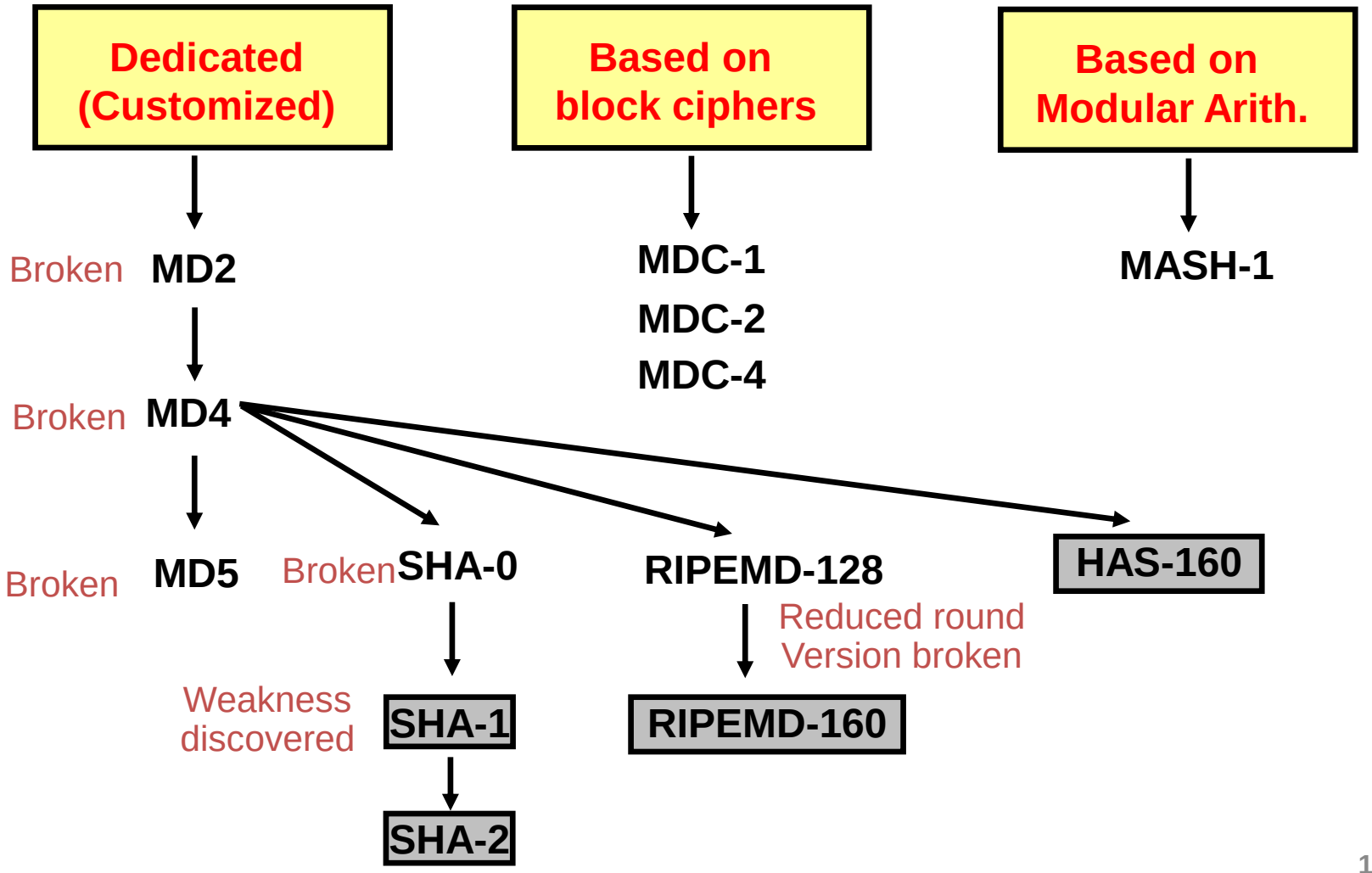
Any collision-resistant compression function *f* can be extended to a collision-resistant hash function *h*

Typical Padding

- **Assume Block size = 512 bits (MD5, SHA-1, RMD160, HAS160 …)**



Hash Function Family



SHA (Secure Hash Algorithm) (1/2)

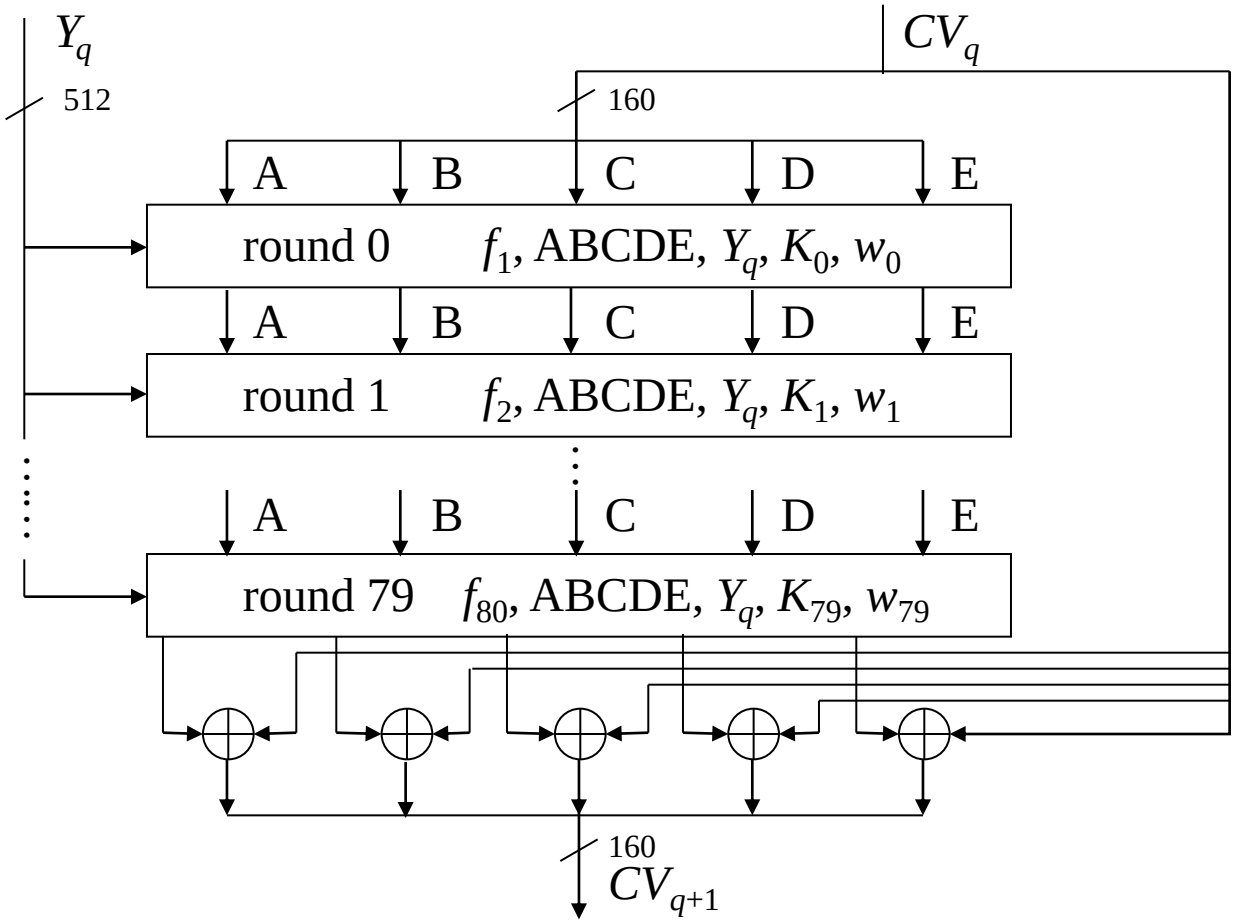
- SHA was designed by NIST (National Institute of Standards and Technology) & NSA (National Security Agency)
- US standard for use with DSA signature scheme
- The algorithm is SHA and the standard is SHS.
- Based on the design of MD4 and MD5 by Rivest@MIT

  - SHA-0: FIPS PUB 180, 1993
  - SHA-1: FIPS Pub 180-1, 1995
    - ✓ bitwise rotation of message schedule of SHA-0 changed
    - ✓ widely-used security applications and protocols such as TLS and SSL, PGP, SSH, S/MIME, and IPsec
  - SHA-2: FIPS Pub 180-2, 2001
    - ✓ SHA-224, SHA-256, SHA-384, and SHA-512
    - ✓ Not so popular as SHA-1

SHA (Secure Hash Algorithm) (2/2)

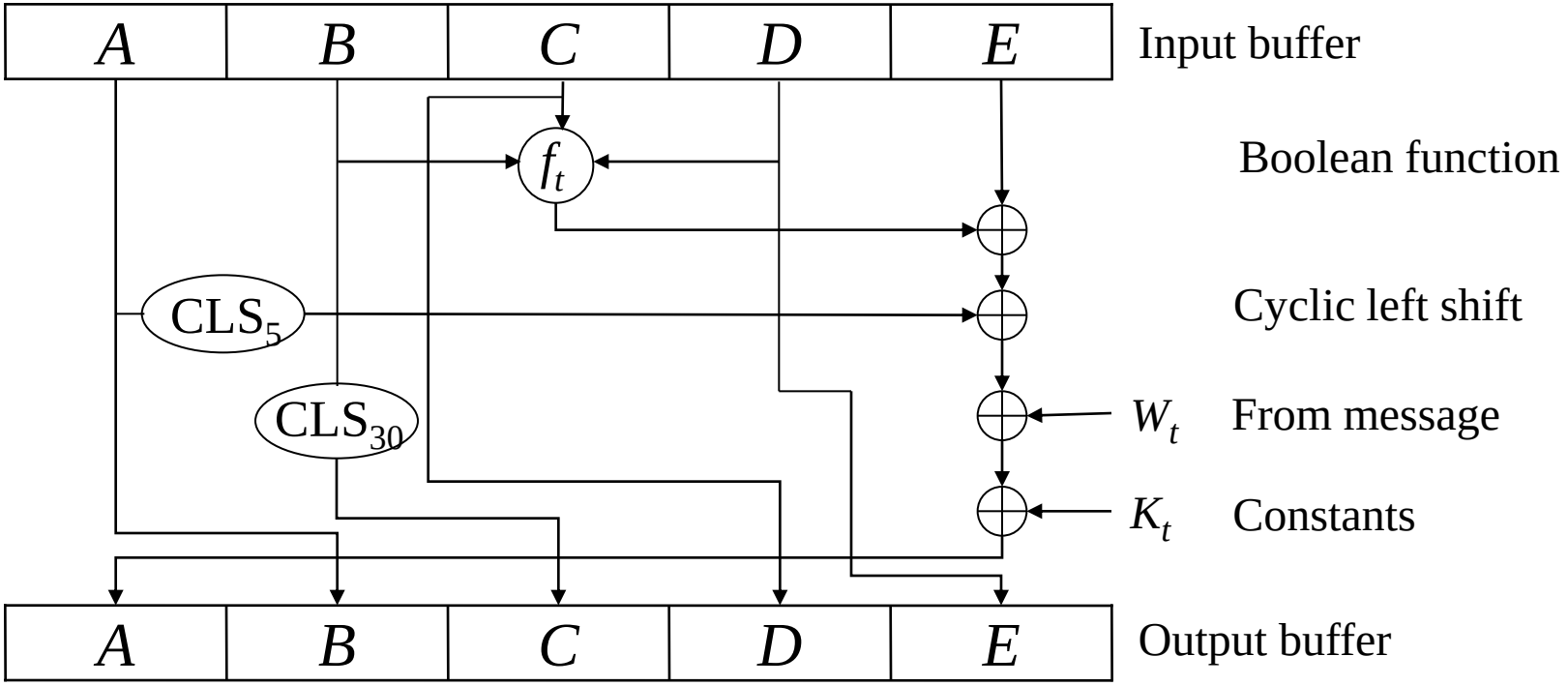
Algorithm and variant		Output size (bits)	Internal state size (bits)	Block size (bits)	Max message size (bits)	Word size (bits)	Rounds	Operation	Collisions Found
SHA-0		160	160	512	$2^{64} - 1$	32	80	+,and,or, xor,rot	Yes
SHA-1		160	160	512	$2^{64} - 1$	32	80	+,and,or, xor,rot	Yes 2^{52} attack (*)
SHA-2	*SHA-256/224*	256/224	256	512	$2^{64} - 1$	32	64	+,and,or, xor,shr,rot	None
	SHA-512/384	512/384	512	1024	$2^{128} - 1$	64	80	+,and,or, xor,shr,rot	None

SHA-1 Overview



http://www.itl.nist.gov/fipspubs/fip180-1.htm

SHA-1 Round Function



SHA-1 Constants & Boolean Functions

Initial values

A = 6 7 4 5 2 3 0 1
B = E F C D A B 8 9
C = 9 8 B A D C F E
D = 1 0 3 2 5 4 7 6
E = C 3 D 2 E 1 F 0

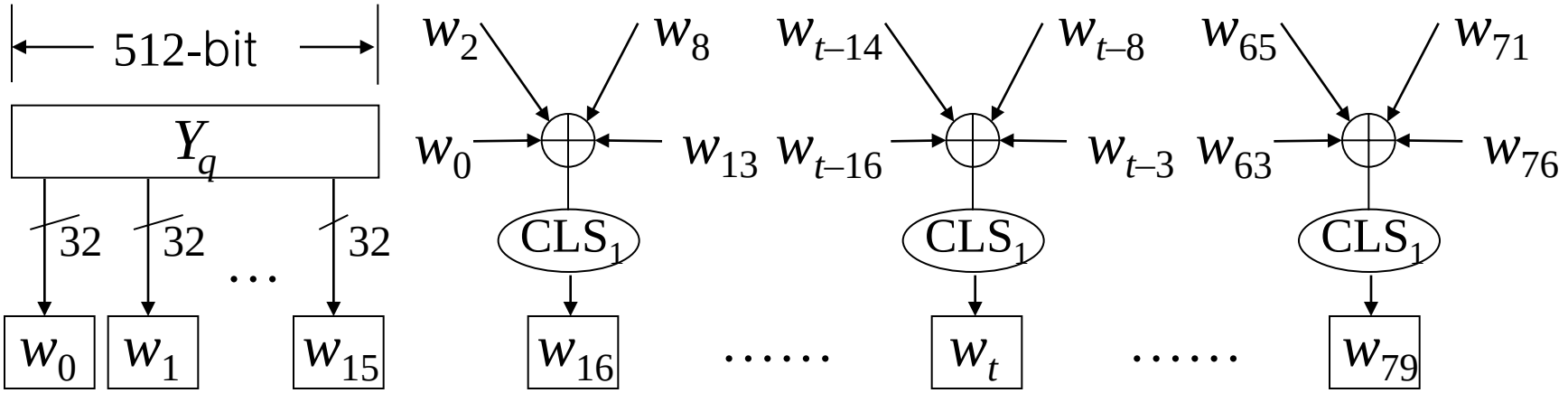
Constants K_t

t = 0 ~ 19	K_t = 5 A 8 2 7 9 9 9
t = 20 ~ 39	K_t = 6 E D 9 E B A 1
t = 40 ~ 59	K_t = 8 F 1 B B C D C
t = 60 ~ 79	K_t = C A 6 2 C 1 D 6

Boolean function f_t

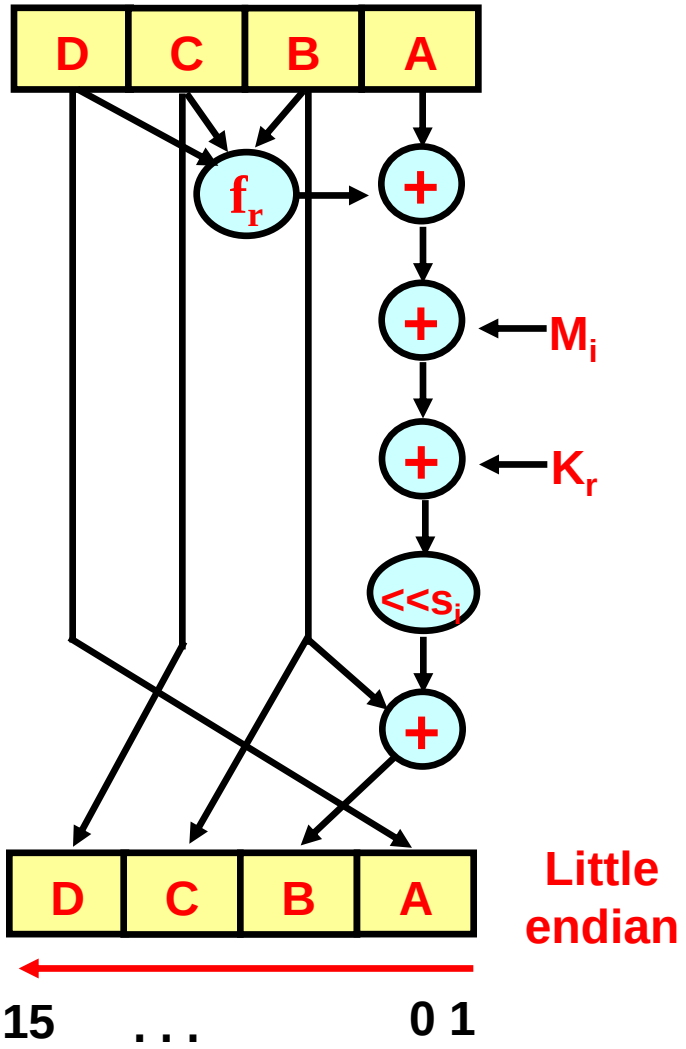
t = 0 ~ 19	$f_t(B, C, D) = B \cdot C + \overline{B} \cdot D$
t = 20 ~ 39	$f_t(B, C, D) = B \oplus C \oplus D$
t = 40 ~ 59	$f_t(B, C, D) = B \cdot C + B \cdot D + C \cdot D$
t = 60 ~ 79	$f_t(B, C, D) = B \oplus C \oplus D$

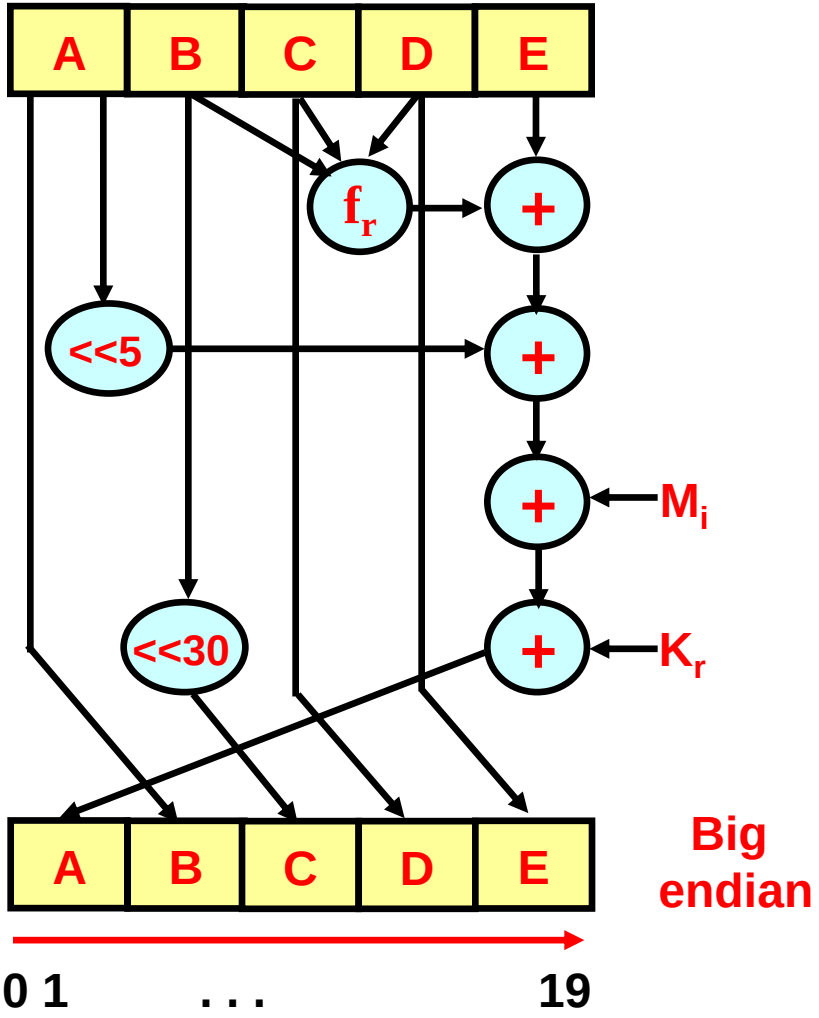
SHA-1 Message Inputs



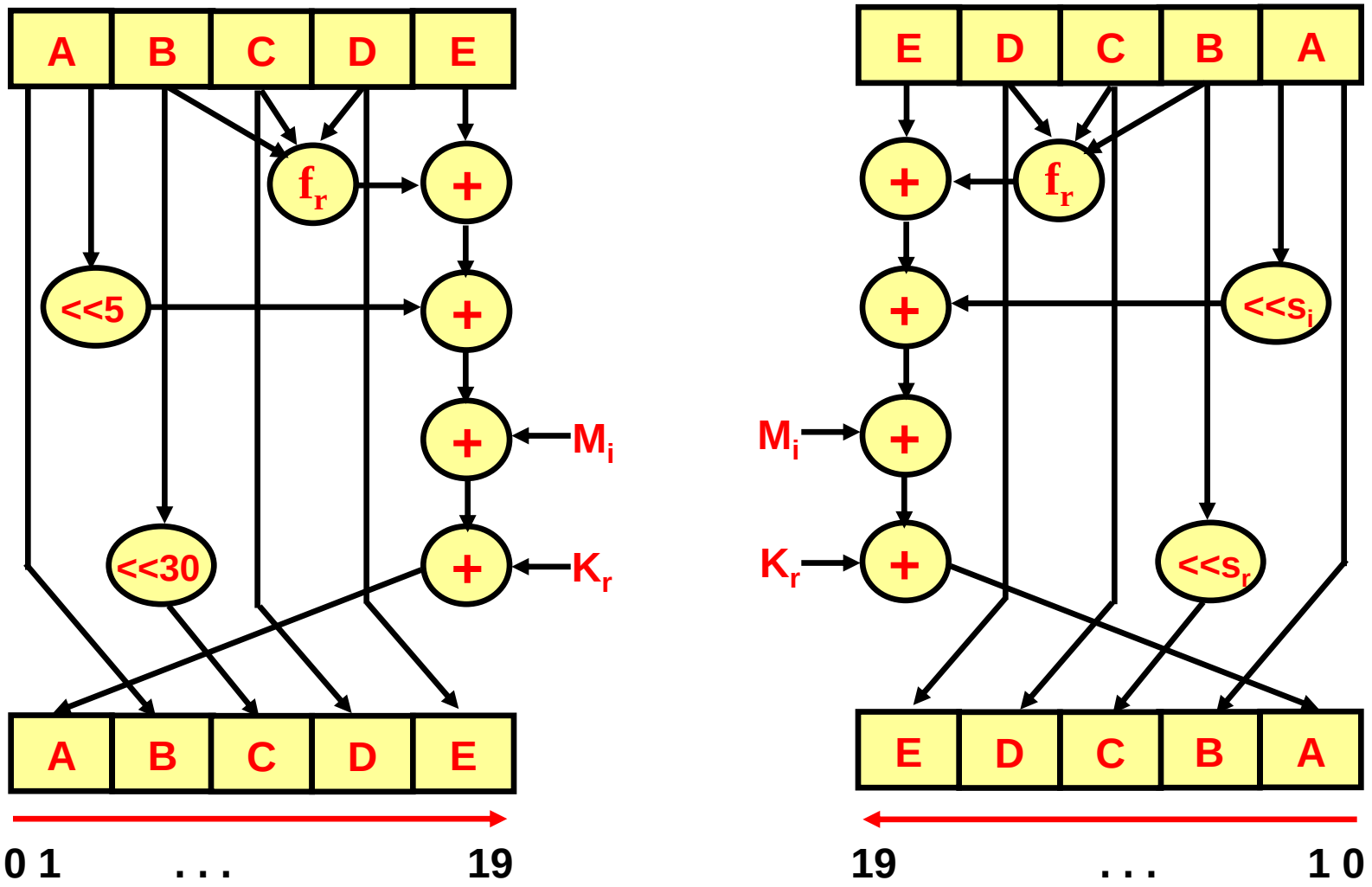
CLS: Cyclic Left Shift

Step Operations of MD5 & SHA-1





Step Operations of SHA1 & HAS160

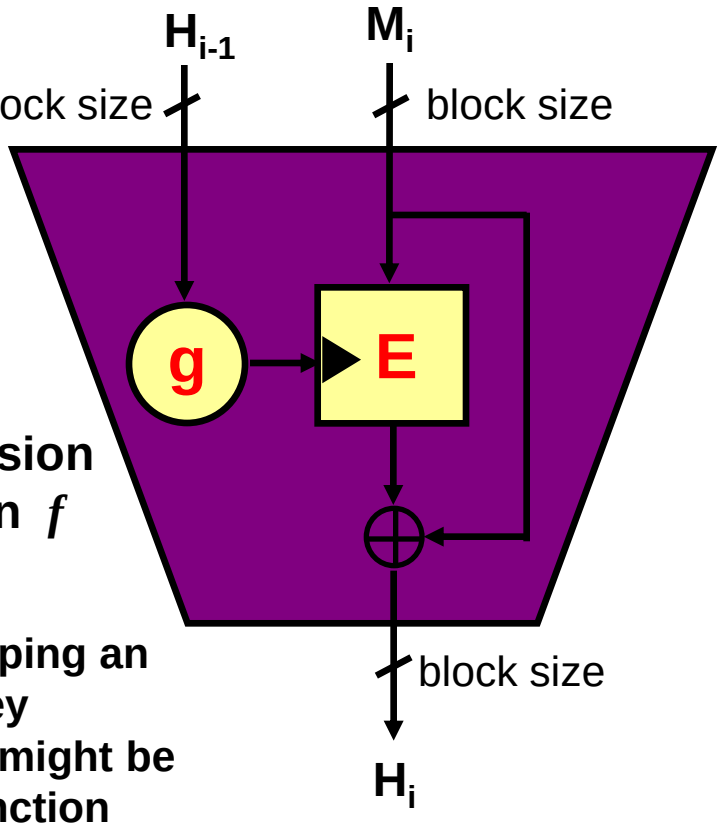


Comparison

Hash Func.	MD5	SHA1	RMD160	HAS160
Digest size(bits)	128	160	160	160
Block size(bits)	512	512	512	512
No of steps	64(4x16)	80(4x20)	160(5x2x16)	80(4x20)
Boolean func.	4	4(3)	5	4(3)
Constants	64	4	9	4
Endianness	Little	Big	Little	Little
Speed ratio	1.0	0.57	0.5	0.94

Hash Ft based on Block Ciphers : MDC1

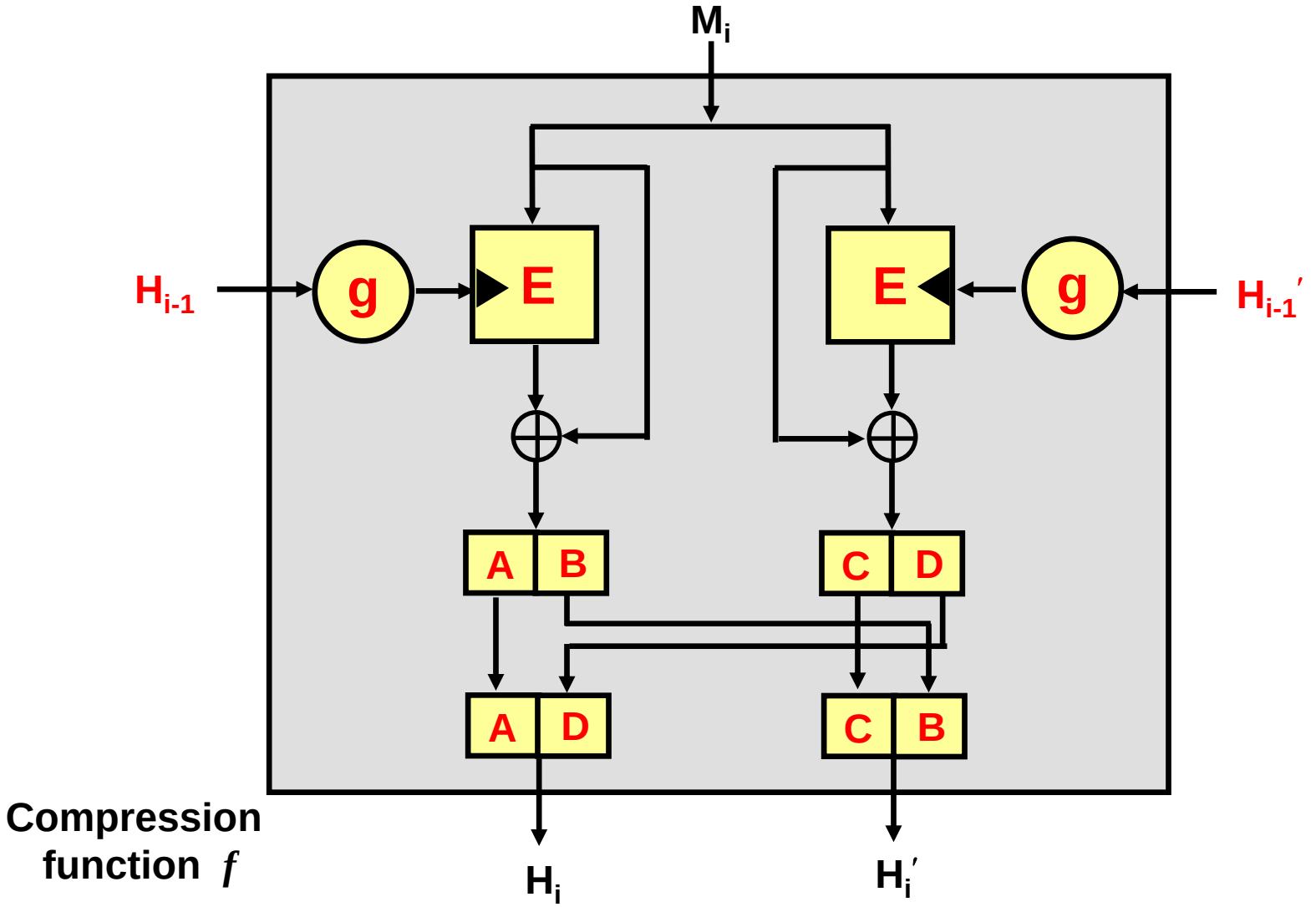
Matyas-Meyer-Oseas Scheme



g: a function mapping an input H_i to a key suitable for E, might be the identity function

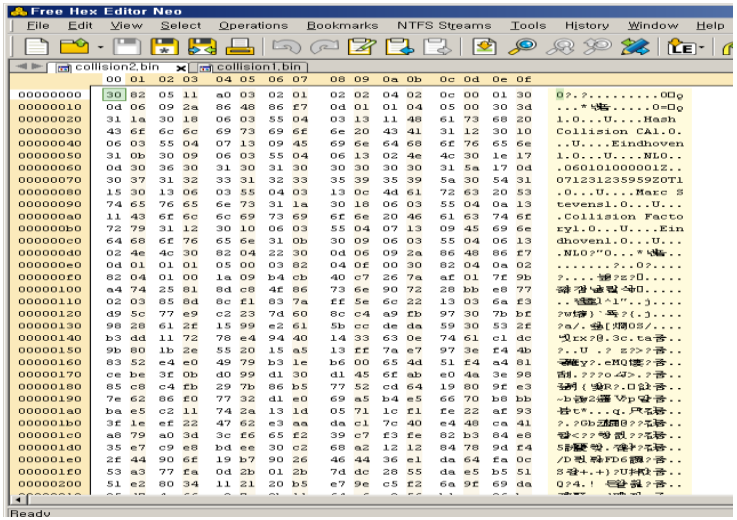
- **Provably Secure under an appropriate black-box model**
- **But produces too short hash codes for use in most applications**

Hash Ft based on Block Ciphers : MDC2

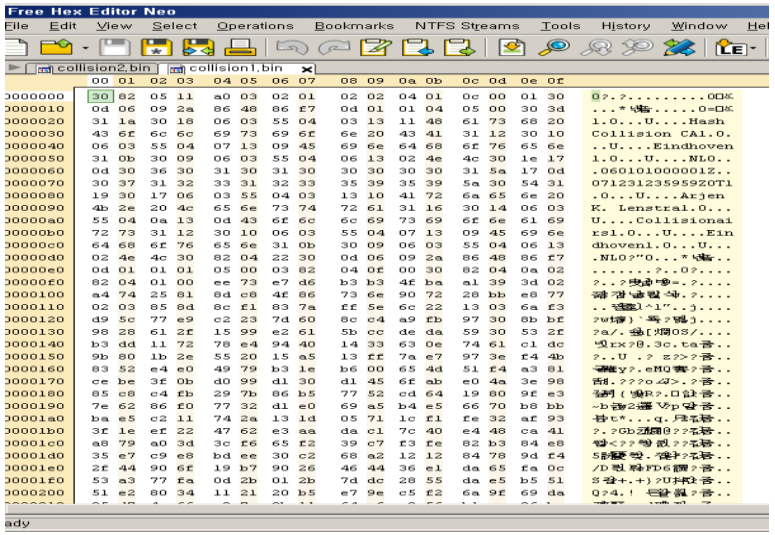


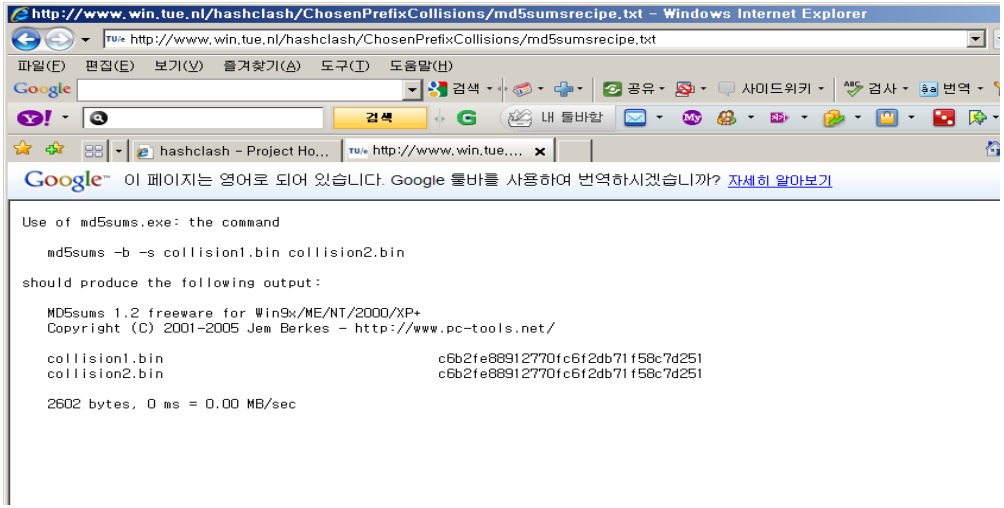
Ex. of MD5 Collisions

Collision1.bin



Collision2.bin





Same MD5 Hashed Value !!

MD5 Collision Attacks

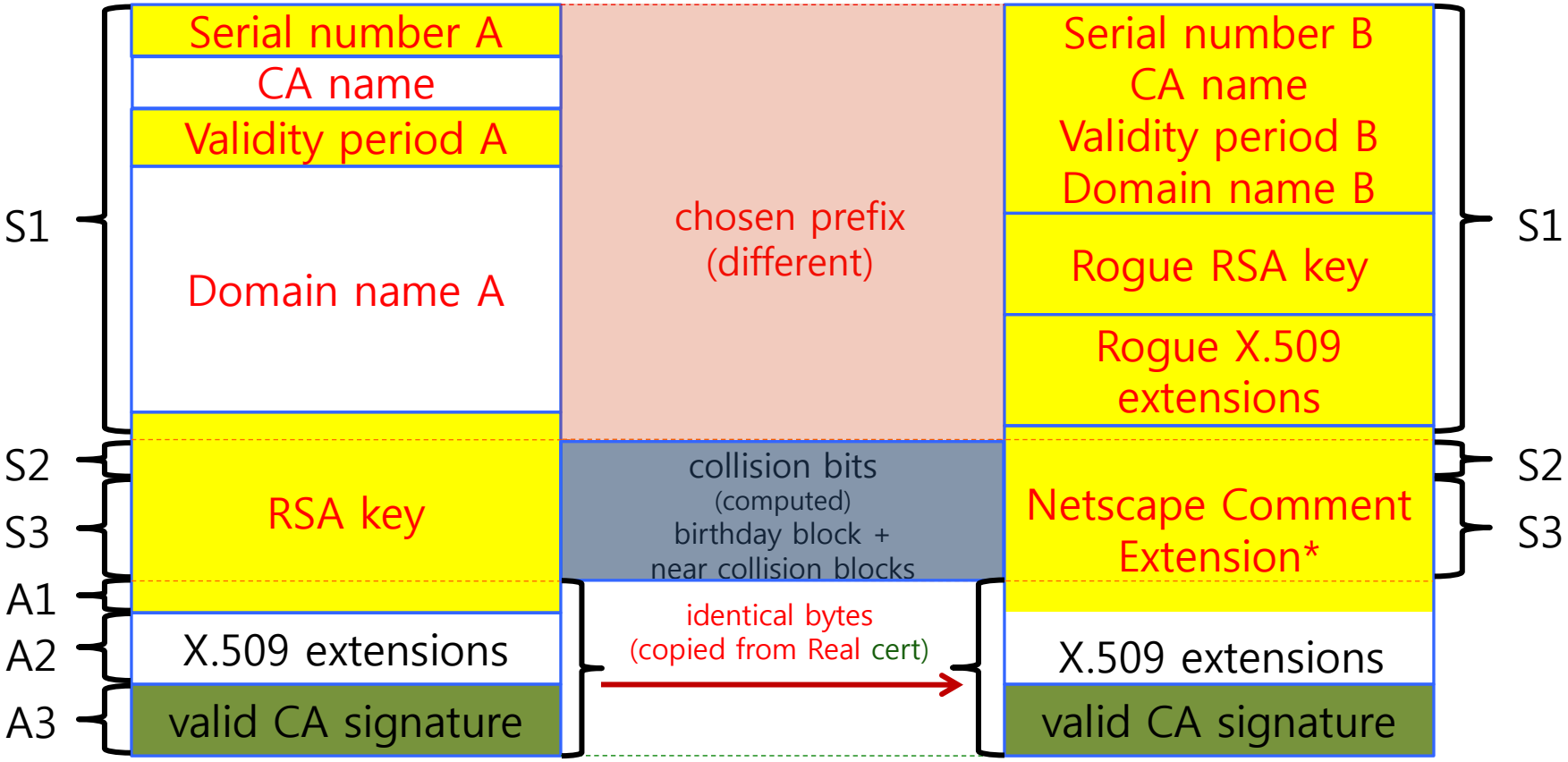
- Colliding valid X.509 certificates
  - Lenstra, Wang, Weger, forged X.509 certificates，http://eprint.iacr.org/2005/067.pdf

    Same owner with different public keys (2048 bits)
  - Stevens, Lenstra, Weger, Eurocrypt 2007

    8192-bit public key (8-block collision)
  - Stevens *etc*. Crypto 2009

    Pass the browser authentication, different owners, different public keys (See next)

X.509v3 Real and Fake Certificates using MD-5



* contents ignored by browsers

SHA-3 Project