## Multi-party Protocol

- (Def.) While keeping each participant's information, $x_{i}$ secret, everyone can learn the result of $\boldsymbol{f}$ (). (If $\boldsymbol{t}$ malicious players exist, we say $\boldsymbol{t}$-secure protocol) -(Privacy) Even if arbitrary subset, $A$ less than the half of an input set behave maliciously, any honest player except $A$ can't know secret $x_{i}$ of $P_{i}$.
-(Correctness) Even if $A$ does any malicious acts, any $P_{j}$ can know the value of $f($ ).

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## ( $\mathrm{n}, \mathrm{k}$ ) Secret Sharing(I) ( $\mathrm{n}>\mathrm{k}$ )

(Step 1) A dealer selects a secret, s(<p:prime) as a constant term and $\boldsymbol{k}$-1 degree random polynomial with arbitrary coefficients as :
$h(x)=s+a_{1} x+a_{2} x^{2}+\ldots+a_{k-1} x^{k-1} \bmod p$
(Step 2) Distributes $n h\left(x_{i}\right){ }^{〔} s(i=1, \ldots, n)$ to a share holder.
(Step 3) When $k$ shadows $K_{1}, K_{2}, \ldots, K_{k}$ among $n$ are given, recover $a_{0}$ by using the Lagrange Interpolation
$h(x)=\sum_{s=1}{ }^{k} K_{i} \prod_{j=1, j \neq s}{ }^{k}\left(x-x_{j}\right) /\left(x_{j}-x_{s}\right) \bmod p$
(Step 4) Recover secret by $h(0)=s$

## ( $\mathrm{n}, \mathrm{k}$ ) Secret Sharing(II)

(Parameter) $n=5, k=3, p=17, s=13$ (secret)
(Polynomial) $h(x)=\left(2 x^{2}+10 x+13\right) \bmod 17$
(Secret sharing) 5 shadows, $K_{1}=h(1)=25 \bmod 17=8, K_{2}=h(2)=7$, $\mathrm{K}_{3}=\mathrm{h}(3)=10, \mathrm{~K}_{4}=\mathrm{h}(4)=0, \mathrm{~K}_{5}=\mathrm{h}(5)=11$
(Recover secret) By using $K_{1}=8, K_{3}=10$, and $K_{5}=11$,
$h(x)=\{8(x-3)(x-5) /(1-3)(1-5)+10(x-1)(x-5) /(3-1)(3-5)+$ 11(x-1)(x-3)/(5-1)(5-3)\} mod 17
$=\left\{8^{*} \operatorname{inv}(8,17)^{*}(x-3)(x-5)+10\right.$ *inv( $\left.-4,17\right)(x-1)(x-5)+11$
*inv(8,17)*(x-1)(x-3)\} mod 17
$=8 * 15(x-3)(x-5)+10^{*} 4^{*}(x-1)(x-5)+11^{*} 15^{*}(x-1)(x-3) \bmod 17$
$=19 x^{2}-92 x+81 \bmod 17=2 x^{2}+10 x+13 \bmod 17$
(Original secret) $h(0)=13$

## ( $\mathrm{n}, \mathrm{k}$ ) Secret Sharing(III)

(Parameter) $\mathrm{n}=3, \mathrm{k}=2, \mathrm{~s}=011$
(Polynomial) irreducible poly over $\mathrm{GF}\left(\mathbf{2}^{3}\right): p(x)=x^{3}+x+1=(1011)$ $->f(\alpha)=0, \alpha^{3}=\alpha+1$
(Secret Sharing) $h(x)=(101 x+011) \bmod 1011$
$K_{1}=\mathrm{h}(001)=(101 * 001+011) \bmod 1011=101+011=110$
$K_{2}=\mathbf{h}(010)=(101 * 010+011) \bmod 1011=001+011=010$
$K_{3}=\mathrm{h}(011)=(101 * 011+011) \bmod 1011=100+011=111$
(Secret Recovering) From given $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$,
$h(x)=[110(x-010) /(001-010)+010(x-001) /(010-001)] \bmod 1011$

$$
=[110(x-010) / 011+010(x-001) / 011] \bmod 1011
$$

Since $011^{-1}=110$, subtraction =addition $->$ bit-by-bit xor
$h(x)=\left[110^{*} 110 *(x+010)+010 * 110 *(x+001)\right] \bmod 1011$
$=[010$ * $(x+010)+111 *(x+001)] \bmod 1011$
$=010 x+100+111 x+111=101 x+011->$ Original secret $: h(0)=011$

## Mental Poker

Non face-to-face digital poker over communication channel.
$\square$ No trust each other.
During setting up protocol, information must be transferred unbiased and fairly. After transfer, validation must be possible.
Expandability from 2 players to $n$ players.

## History of Mental Poker

- SRA('79) : Using RSA
- Liption/Coppersmith('81) : Using Jacobian value
- GM(‘82) : Using probabilistic encryption
- Barany \& Furedi ('83) : Over 3 players
- Yung('84)
- Fortune \& Merrit('84) : Solve player’s compromise
- Crepeau ('85) : Game without trusted dealer
- Crepaeu('86) : ZKIP without revealing strategy
- Kurosawa('90) : Using r-th residue cryptosystems
- Park('95) : Using fault-tolerant scheme


## Basic Method

## $\square$ A (Dealer) shuffles the card.

- B selects 5 cards from A.
$\square$ (Problem)
- A can know B's selection.
$-A$ is in advantage position than $B$.
$\square$ (Solution)


## Use cryptographic protocols.

## Mental Poker by SRA(I)

(Preparation) $A$ and $B$ (dealer) prepare public and private key pairs $\left(P_{A}, S_{A}\right)$ and $\left(P_{B}, S_{B}\right)$ of RSA cryptosystem respectively.
(Step 1) Using B's public key, he posts all 52 encrypted cards $E\left(P_{B}\right.$, $m_{j}$ ) in the deck.
(Step 2) A selects 5 cards in the deck and sends them to $B$.
(Step 3) B decrypts $D_{B}\left(S_{B}, E\left(P_{B}, m_{j}\right)\right)=m_{i}$ using his secret key and keep them as his own cards.
(step 4) A selects 5 cards from the remaining 47 cards and encrypts using his public key $E\left(P_{A}, E\left(P_{B}, m_{j}\right)\right)$ and sends them to $B$.
(step 5) $B$ decrypt 5 cards using $B$ 's secret key $D\left(S_{B}, E\left(P_{A}, E\left(P_{B}, m_{j}\right)\right)\right.$ ) and send $E\left(P_{A}, m_{j}\right)$ to $A$
(step 6) Using A's secret key, A decrypts $E\left(P_{A}, m_{j}\right)$ and keeps them as his cards.
Winner Decision : Reveal his own (opened) cards to counterpart
Validation : Reveal his secret cards to counterpart

## Mental Poker by SRA(II)


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## Electronic Vote

$\square$ Yes-No (Binary) Vote

- While keeping each voter's vote secret ( $x_{i}$ ), compute only total sum ( $T=x_{1}+x_{2}+\ldots+x_{n}$ )
- Malicious players among $n$ exist (interruption etc.)
- $t$-secure multiparty protocol
- Basic tool
- VSS (Verifiable Secret Sharing) - OT (Oblivious Transfer)


## Requirement of E-vote

- Privacy : keeping each vote secret
- Unreusability : prevent double voting
- Fairness : if interruption occurs during voting process, it doesn't affect remaining voting
- Eligibility : only eligible voter can vote
$\square$ Verifiability : can't modify voting result
- Soundness : preventing malicious acts
$\square$ Completeness : exact computation


## Cryptographic tool for e-vote



## Implementation Methods

- Using RSA
- Koyama (NTT), Meritt(America), Assuming trustful center
- Using r-th residue cryptosystem
- Small-scale vote by Kurosawa(TIT)
- Using Blind Signature
- Large scale voting,
- Administrator, Tally,
- Application of multiparty protocol
- Benaloh(America), Iverson(Norway) etc
- Keeping voter's vote secret, small-scale yes-no vote
- Using Anonymous Channel
- Chaum(Netherland), Ohta/Fujioka(NTT), Sako(NEC), Park(Korea) etc
- Unlinking vote and voting, suitable for large scale voting
- Others
- multi-recastable ticket
- receipt-freeness: prevent buying vote, coercion


## E-vote by RSA

voter $i$
$v_{i}=$ contents of voting

(Voting Procedure)
(Step 1) voter $i$ casts his vote by computing $C_{i}=E_{A}\left(D_{i}\left(E_{T}\left(v_{j}\right)\right)\right)$
(Step 2) After checking voter's identification, Admin $A$ sends $Z_{i}=E_{T}\left(D_{A}\left(E_{i}\left(D_{A}\left(C_{j}\right)\right)\right)=E_{T}\left(D_{A}\left(E_{T}\left(v_{j}\right)\right)\right)\right.$ to $T$.
(Step 3) T make $D_{T}\left(E_{A}\left(D_{T}\left(Z_{j}\right)\right)\right)=v_{i}$ to be public.
${ }^{*} v_{i}=D_{T}\left(E_{A}\left(E_{T}\left(D_{A}\left(E_{i}\left(D_{A}\left(E_{A}\left(D_{T}\left(D_{i}\left(E_{T}\left(v_{i}\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)$-> reblocking problem

## E-vote by PKC

$\square$ A voter sends his vote by encrypting center's public key.

- Center decrypts each votes by its secret key and accumulate each vote.
$\square$ (Problem)
- Revealing of voter's privacy
- Malicious act of centers : post it in the bulletin board


## $r$-th residue

(Def.) Given integer $n$, an integer $z$ is called as $r$-th residue mod. $n$ iff $\exists$ some integers $x$ s.t. $z=x^{r} \bmod n$.
(Notation) $Z_{n}{ }^{r}$ : set of $r$-th residues mod $n$ which are relatively prime to $n, \quad Z_{n}{ }^{r}$ : set of $\boldsymbol{z}$ in $Z_{n}{ }^{*}$ which are not $r$-th residues $\bmod n$
(Lemma)

1. $Z_{n}{ }^{r}$ is a subgroup of $Z_{n}{ }^{*}$
2. Given a fixed $r$ and $n$, every integer $z$ in $Z_{n}{ }^{r}$ has the same number of $r$-th roots.
3. If $r$ and $\varphi(n)$ are relatively prime, every integer $z$ in $Z_{n}{ }^{*}$ is an $r-$ th residue $\bmod n\left(i . e ., Z_{n}{ }^{r}=Z_{n}{ }^{*}\right.$ ) and $r$-th root of $z$ is given by $z^{A} \bmod n$ where $A$ satisfying $A r-B \varphi(n)=1$.

## $r$-th residue cryptosystem(I)

$\square$ secret key: primes $p, q$
$\square$ public key : $N(=p q), y$
$\square$ message : $m(0 \leq m<r), r\left({ }^{*}\right)$ : random number
a encryption [KKOT90]

- $E(m)=y^{m} x^{r} \bmod N(x$ : random number)
$-E(m) \cdot E(n)=y^{m} x_{1}{ }^{r} \bullet y^{n} x_{2}^{r} \bmod N$ $=y^{(m+n)}\left(x_{1} x_{2}\right)^{r} \bmod N=y^{(m+n)} z^{r} \bmod N$
Thus, $E(m+n)=E(m) E(n) z^{r} \bmod N$ for some $z$
(additive homomorphism)
(*) If $r=2[G M 82],(y / p)=(y / q)=-1$.
prime $r$ [CF85][BY85], $r|p-1, r| / q-1, y$ is $r$-th non-residue.


## $r$-th residue cryptosystem(II)

## Decryption

$\square y^{j} \notin B_{N}(r), 1 \leq j<r, B_{N}(r)=\left\{w \mid w=x^{r} \bmod N, x \in Z_{N}{ }^{*}\right\}$
$-\operatorname{gcd}(p-1, r)=e_{1}, \operatorname{gcd}(q-1, r)=e_{2}$

- $r=e_{1} e_{2}$ if $r$ is odd, $r=\left(e_{1} e_{2}\right) / 2$ if even
$-\operatorname{gcd}\left(\mathrm{e}_{1}, \mathrm{e}_{2}\right)$ is 1 if $r$ is odd, 2 if even
- $(y / N)=1$ if $r$ is even.
- Under mod p $\{E(m)\}^{(p-1) / e_{1}}=\left(y^{m} x^{r}\right) y^{(p-1) / e_{1}}=\left(y^{(p-1) / e_{1}}\right)^{m}\left(x^{r / e 1}\right)^{(p-1)}$ $=\left(y^{(p-1) / e_{1}}\right)^{m}$
- Similarly under mod $q,\{E(m)\}^{(q-1) / e_{2}}=\left(y^{(q-1) / e_{2}}\right)^{m}$
- Thus, for $0 \leq i<r$, compare $\{E(m)\}^{(p-1) / e_{1}}$ and $\{E(m)\}^{(q-1) / e_{2}}$ with $\left(y^{(p-1) / e_{1}}\right)^{i}$ and $\left(y^{(q-1) / e_{2}}\right)^{i}$ respectively
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## E-voting(1) - 1 center -

## Basic Protocols

(1) Center publishes $r$-th residue cryptosystem's public key ( $N, y$ ). (\# of voters, $h$ are less than $r$ )
(2) Each voter $i$ encrypts his vote depending on $m_{i}=0$ or 1 and sends $E\left(m_{i}\right)=y^{m_{i}} X_{i}^{r} \bmod N$ to a center ( $x_{i}$ is a large random number.)
(3)Center publish $M=m_{1}+m_{2}+\ldots+m_{h}$ to the public

## E-voting(2) - 1 center -

(1) Center shows that " $(N, y)$ is public key information of r-th residue cryptosystem in ZKIP"
(2) Each voters show that "The plaintext of $E\left(m_{j}\right)$ is $m_{i}=0$ or 1 in ZKIP" (cryptographic capsule)
(3) Center shows that "In order that $\mathrm{E}\left(\mathrm{m}_{1}\right)$ $E\left(m_{h}\right)=y^{M} x^{r} \bmod \mathbf{N}\left(\right.$ where $M=m_{1}+\ldots+$ $\left.m_{h}\right)$, prove that $z=y^{M} x^{r} \bmod N\left(x=x_{1} \ldots x_{h}\right)$ in ZKIP.


## Multiple centers

- Voter i
$-m_{i}=m_{i 1}+\ldots+m_{i n} \bmod r$
- $E\left(m_{i 1}\right)$-> center $1, \ldots$
- $E\left(m_{i n}\right)$-> center $n$
- Center $\boldsymbol{j}$
- $E_{j}\left(M_{i j}\right)$
- $E_{j}\left(M_{2 j}\right)$

Publish $M_{j}=M_{1 j}+\ldots+M_{k j}$

- $E_{j}\left(M_{k j}\right)$
$\square$ Voting result
$-M=M_{1}+\ldots+M_{n}$


## Problems of multiple centers

- If a center fail, voting fails too.
$\rightarrow$ Introducing Secret Sharing Scheme.
- If a voter can play as a center, we don't need a center.


## E-voting using SSS

- Voter i
$-f_{i}(x)=m_{i}+a_{1} x+\ldots+a_{k-1} x^{k-1}$
- $E_{1}\left(f_{i}(1)\right)$ : to center $1, E_{2}\left(f_{i}(2)\right)$ : to center $2, \ldots, E n\left(f_{i}(n)\right)$ : to center $\mathbf{n}$
- If only $k$ centers cooperate, we can know $m_{i}$.
- Center j publishes $M_{j}=f_{1}(j)+\ldots+f_{n}(j)$
$-f(x)=f_{1}(x)+\ldots+f_{n}(x)$
$=\left(m_{1}+\ldots+m_{k}\right)+a_{1}^{\prime} x+\ldots a_{k-1}^{\prime} x^{k-1}$
,$f(j)=M_{j}$
- Even if ( $n-k$ ) centers fail, if we know $\boldsymbol{k} \boldsymbol{M}_{\boldsymbol{j}}$, then recover $\left(m_{1}+\ldots+m_{k}\right)$.


## Verification

- Voter i
$f_{i}(x)=m_{i}+a_{1} x+\ldots+a_{k-1} x^{k-1}$ $y_{1}=E_{1}\left(f_{i}(1)\right):$ to center 1
...
$y_{n}=E_{n}\left(f_{i}(n)\right):$ to center $n$
To show that $\left(y_{1}, \ldots, y_{n}\right)$ is computed by above equations in ZKIP -> VSS (Benaloh'86)


## Reminding ZKIP

- If there is a secure probabilistic encryption, then every language in NP has ZKIP in which the prover is a probabilistic polynomial-time machine that gets an NP proof as an auxiliary input [GMW85] .
- An encryption system secure as in [GM84] is a probabilistic poly-time algorithm $f$ that on input $x$ and internal coin tosses $r$, outputs an encryption $f(x, r)$. Decryption is unique : that is $f(x, r)=f(y, s)$ implies $x=y$.


## VSS(I)

SS+ZKP
(Purpose) To show a dealer behaves in a right way, (i.e. any number of more than $\boldsymbol{k}$ shareholders can reveal same secret in ZKIP).
(1) A dealer encrypt a secret, $m$ to $c(m)$ and send it to $n$ shareholders.
(2) Using SSS, a dealer sends $f(j)(j=1, \ldots, n)$ to each shareholder $j$.
(3) A dealer show each shadows was constructed by the above procedure by using ZKIP
(Tools) Checking each shadow in a correct way is NP problem. If there is 1 -way function, there always exist ZKPS to prove this.

## VSS(II)

- (Assumption) arbitrary 1-way permutation
$\square(k, n)$ secret $s \in Z_{p}$
- [Preparation] Sender $\boldsymbol{k}$-1 degree random polynomial over $Z_{p}{ }^{*}$ and computes $n$ shares.
- Senders encrypt $i$-th piece with user i's PKC.
- Sender provide each receiver with ZKP that encrypted messages correspond to the evaluation of a single polynomial over $Z_{p}{ }^{*}$ and applying $f$ to the constant term of this polynomial yield $s$.


## VSS using $r$-th residue cryptosystem(I)

(step1) A dealer encrypts the $i$-th shareholder's secret, $s_{i}=f(i)$ by using $r$-th residue cryptosystem, $z_{i}=y_{i}{ }^{\mathbf{s i}_{i}} \mathbf{x}_{i}{ }^{r} \bmod N_{i}$ and makes it public. The $i$-th shareholder decrypts this and recover his secret information, $\mathbf{s}_{\mathrm{i}}$.
The following is considered as ZKIP about
$L=\left\{z_{1}, \ldots, z_{n} \mid z_{i}=y_{i}{ }^{s i} x_{i}^{r} \bmod N_{i}, s_{i}=f(i)\right\}$. Repeat steps (2)~ (4) $t$ times, $t=$ number of bits in N .
(step2) A dealer selects random polynomial $f^{\prime}$ of degree ( $k-1$ ) and computes the same as (step 1). i.e., a dealer
computes the $i$-th shareholder's secret, $s_{i}^{\prime}=f^{\prime}(i)$ by using r-th residue cryptosystem, $z_{i}^{\prime}=y_{i}{ }^{s^{\prime}} \mathbf{x}_{i}^{\prime}{ }^{r} \bmod N_{i}$. The i-th share holder decrypts this and recovers his secret information $\mathbf{s}^{\prime}{ }_{i}$

## VSS using $r$-th residue cryptosystem(II)

(step 3) The shareholders send $e=1$ or 0 to a dealer. (All shareholders agree the value of $e$ ).
(step 4) If $e=0$, the dealer reveals all $s_{i}^{\prime}$ and $x_{i}^{\prime}$ and shows $f^{\prime}$ has degree of $(k-1)$. If $e=1$, the dealer shows all $t_{i}$ and $w_{i}$ satisfying $z_{i} z_{i}^{\prime}=y_{i}^{t_{i}} w_{i}^{r} \bmod N_{i}$ and $f+f^{\prime}$ has degree of $(k-1)$.
(Example) A voter sends his vote to $\boldsymbol{n}$ centers, it is hard to reveal his secret voting without collaborating more than $k$ centers.

## OT(Oblivious Transfer)(I)

(Purpose) While keeping secret, sending the corresponding information.
(Ex) OT : Alice has a secret bit, b. At the end of protocol, one of the following two events occurs, each with probability 1/2.
(1) Bob learns the value of $b$.
(2) Alice gains no further information about the value of $b$ (other than what Bob knew before the protocol)
[Result] If there exists PKC, feasible to construct OT[EGL85]
[Application] electronic contract signing, multi-party protocol, etc.

## OT(Oblivious Transfer)(II)


$B$ can derive $m_{b}$,but can't derive $m_{b \oplus 1}$ because it is equivalent to derive $D_{S A}\left(E_{P A}(x)+r_{b}-r_{b \oplus 1} \bmod N\right)$ which is hard to solve PKC itself.

## OT (III)

[1-2 Oblivious String Transfer]
Alice has 2 strings, $S_{0}$ and $S_{1}$. Bob has a selection bit, s. At the end of protocol, the following three conditions hold.
(1) Bob learns the value of $\mathrm{S}_{s}$.
(2) Bob gains no further information about the value of $\mathrm{S}_{1-\mathrm{s}}$.
(3) Alice learns nothing about the value of $s$.

Alice has 2 secret strings. Bob select exactly one of them, and Alice doesn't know which secret Bob selected.
[ Oblivious Circuit Evaluation] Alice has some secret, $i$, and Bob has some secret, j. Both agreed on some circuit $f$. At the end of protocol, the following three conditions holds.
(1) Bob learns the value of $f(i, j)$.
(2) Bob learns no further information about $j$ (other than that revealed by knowing $i, f(i, j)$.
(3) Alice learn nothing about $i$ or $f(i, j)$.

## Anonymous Channel(I)

(Def 1) A channel is a set of probabilistic polynomial time Turing machines ( $P_{1}, \ldots, P_{n}, S_{1}, \ldots, S_{n}$ ) together with a public board. $P_{i}$ is called a sender, $S_{i}$ is called a shuffle machine agent. $P_{i}$ or $S_{i}$ is called a player.
(Def 2) Let $m_{i}$ be input of $P_{i}$ and OUT $=\left\{o_{1}, \ldots, o_{n}\right\}$ be the final list of public board.A channel is called an anonymous channel if the following conditions hold.
[Completeness] If every player is honest, $\left\{\mathrm{o}_{1}, \ldots, \mathrm{o}_{\mathrm{n}}\right\}=\left\{\mathrm{m}_{1}, \ldots, \mathrm{~m}_{\mathrm{n}}\right\}$.
[Privacy] For any $i$, the correspondence between $P_{i}$ and $m_{i}$ is kept secret.

An election scheme is an anonymous channel with the following condition.
[Verifiability] If $\left\{\mathrm{o}_{1}, \ldots, \mathrm{o}_{\mathrm{n}}\right\} \neq\left\{\mathrm{m}_{1}, \ldots, \mathrm{~m}_{\mathrm{n}}\right\}$, every $\mathrm{P}_{\mathrm{i}}$ can detect this fact with overwhelming probability.

## Anonymous Channel(II)

## Simple Mix Anonymous Channel

(Preparation)Sender : $A_{1}, \ldots A_{n}$, Receiver: $B_{i}, B_{i}$ 's public key : $E_{B i}$, Role of shuffle agent $S_{i}$ : decrypting each sender's encryption, removing its random part, and sorting alphabetical order then output $S_{i}$ 's public key : $\mathrm{E}_{\mathrm{i}}$
(Purpose) Each sender doesn't know the corresponding information of message, $\mathrm{m}_{\mathrm{i}}$.
(step 1) Each $A_{i}$ chooses a random number $R$ and writes $C_{i}=E_{1}\left(R^{\circ} B_{i}{ }^{\circ} E\right.$ $\mathrm{Bi}_{\mathrm{i}}\left(\mathrm{m}_{\mathrm{i}}\right)$ ) on the public board.
(step 2) $S_{1}$ decrypts and throws away $R$, and then writes $\left\{B_{i}{ }^{\circ} E_{B i}\left(m_{i}\right)\right\}$ on the public board in lexicographical order.
This gives that everyone except $S_{1}$ can't tell the correspondence between $\left\{A_{i}\right\}$ and $\left\{B_{i}\right\}$.

If a Mix is dishonest, it will be big problem.!

## E-vote by anonymous channel(I)

(To prevent malicious acts of Mix)
[Registration phase]
(step 1) Each $P_{i}$ chooses $\left(K_{i}, K_{i}{ }^{-1}\right)$ where $K_{i}$ is public key and $K_{i}{ }^{-1}$ is its secret key. $P_{i}$ writes $E_{1}\left(R_{1}{ }^{\circ} E_{2}\left(R_{2} \ldots E_{k}\left(R_{k}{ }^{\circ} K_{i}\right) \ldots\right)\right)$ on the public board with his digital signature.
(step 2) The $k$ MIXes anonymous channel shuffles $\left\{K_{i}\right\}$ in secret.
(step 3) $\mathrm{S}_{\mathrm{k}}$ writes $\mathrm{K}_{\mathrm{i}}$ on the public board in lexicographical order.
Let the list be ( $\mathrm{K}_{1}, \mathrm{~K}_{2}, \ldots$ ).
[Claiming phase]
(step 4) Each $P_{i}$ checks that his $K_{i}$ exists in the list. If not, $P_{i}$ objects and election stops. If no objects in some period of time, goto the next phase.

## E-vote by anonymous channel(II)

[Voting phase]
(step 5) Each $P_{i}$ writes $E_{1}\left(R_{1}{ }^{\circ} E_{2}\left(R_{2} \ldots E_{k}\left(R_{k}{ }^{\circ}\left(K_{i}{ }^{\circ} K_{i}{ }^{-1}\left(V_{i}{ }^{\circ} 0^{\prime}\right)\right)\right) \ldots\right)\right.$ ) on the public board with his digital signature.
(step 6) After the voting is over, the $\mathbf{k}$ MIXes anonymous channel shuffles $K_{i}{ }^{\circ} K_{i}{ }^{-1}\left(V_{i}{ }^{\circ} 0^{\prime}\right)$ in secret.
(step 7) $\mathrm{S}_{\mathrm{k}}$ writes $\mathrm{K}_{\mathrm{i}}{ }^{\circ} \mathrm{K}_{\mathrm{i}}{ }^{-1}\left(\mathbf{V}_{\mathrm{i}}{ }^{\circ} \mathbf{0}^{\prime}\right)$ on the public board in lexicographical order. Let the list be ( $u_{1}{ }^{\circ} v_{1}$ ), $\left(u_{2}{ }^{\circ} v_{2}\right), \ldots$
(step 8) Everyone checks that $u_{i}=K_{i}^{\prime}$ and $u_{i}\left(v_{i}\right)={ }^{*} \ldots{ }^{*} 0^{\prime}$ for each $i$. If the checks fails, stop.
(step 9) It is easy for everyone to obtain $\left\{\mathrm{V}_{1}, \ldots, \mathrm{~V}_{\mathrm{n}}\right\}$.

## Other e-voting scheme

## Receipt-free

$\square$ Universal Verifiability
-Local verifiability
-Universal verifiability
$\square$ Mix-net based e-voting

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