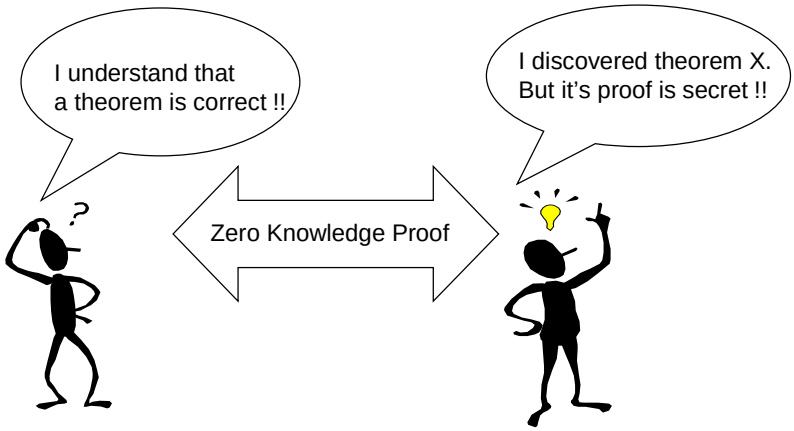
Cryptographic Protocols(I)

- 1976 : Birth of concepts of PKC
- 1978 : Birth of RSA
  - New applications compared to traditional concepts
    - ✓ Digital Signature
    - ✓ Coin Flipping
    - ✓ Mental Poker
    - ✓ Contract Signing
    - ✓ Electronic Voting
    - ✓ Comparison of Richness

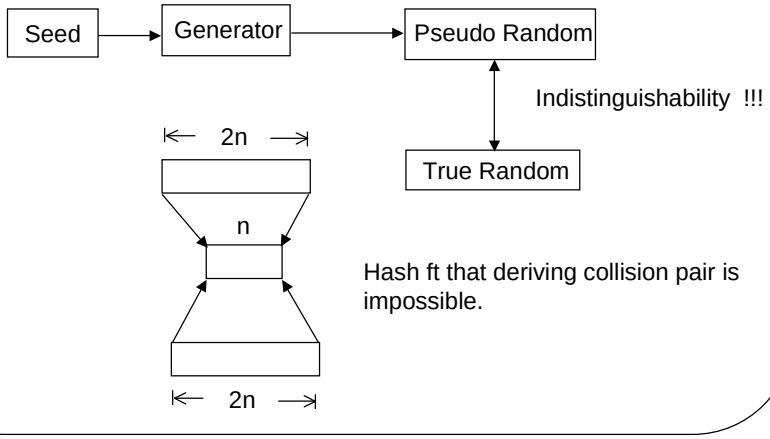
Cryptographic Protocols(II)

- 1978 - 1984
  - A variety of PKCs
  - Research on various cryptographic protocols
- 1985
  - ZKIP (Zero Knowledge Interactive Proof)
  - Authentication
  - Multiparty Protocol
  - Proof of NP-complete problem

Cryptographic Protocols(III)



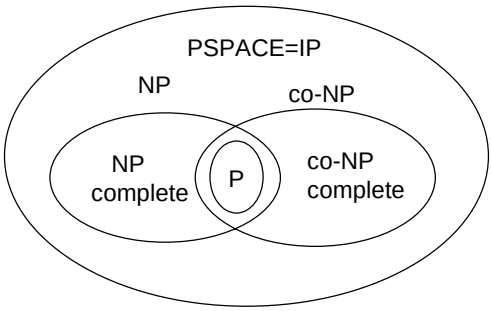
Application of PKC



Hash ft that deriving collision pair is impossible.

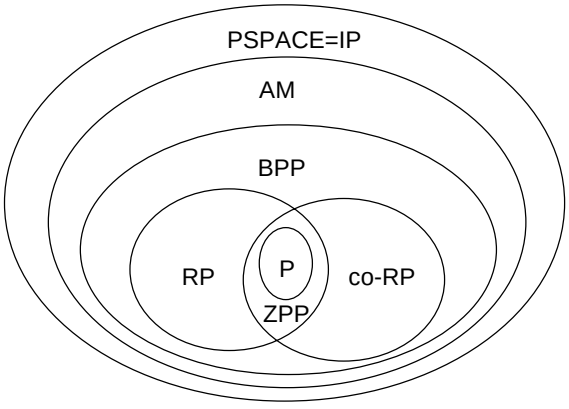
Complexity Class(I)

Language L={0,1}*: infinite set of elements with various input size
Uniform Model : Turing Machine (computer algorithm)
Non-uniform Model : Circuit model (VLSI)
P : Deterministic poly, NP : Non deterministic Poly

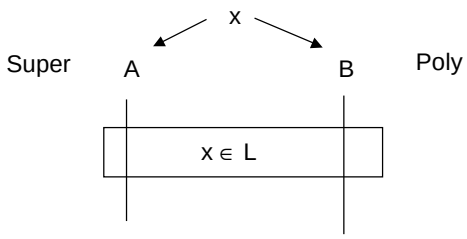


Complexity Class(II)

Allows random coin -> error



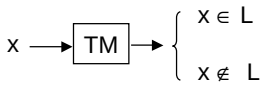
Computation & Proof(I)



For B
- no help : P, BPP
- 1-way proof : NP
- interactive proof : IP
- \+
- zero knowledge = ZKIP

Computation & Proof(II)

$L \in P$

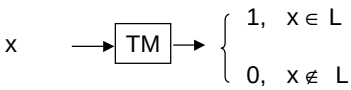


Prob. of error =0

$L \in BPP$

Poly-time

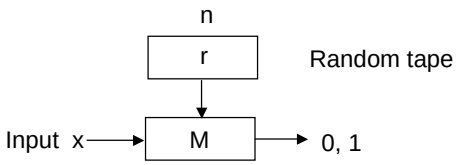
Random tape



Completeness $x \in L$ Prob(TM(x)=1) $\geq 2/3$

Soundness $x \notin L$ Prob(TM(x)=0) $\geq 2/3$

BPP(I)



r	r_1 ...	r_k	r_{k+1} ...	r_{2^n}
M(x,r)	1 …	1	0 …	0

$$\text{Prob}(M(x) = 1) = k/2^n \begin{cases} > 1 - \varepsilon & \text{if } x \in L \\ < \varepsilon & \text{if } x \notin L \end{cases}$$

$$\text{Prob}(M(x)=0) = (2^n - k) / 2^n$$

M(x) : random variable

BPP(II)

- **Example of BPP**

L = { *p* | *p* = prime }

Probabilistic prime test by Solovay-Strassen

- ✓ **gcd (*a*,*p*) =1 (1)**
- ✓ **$(a|p) = a^{(p-1/2)} \bmod p$ (2)**

If $p \in L$, eqs (1) and (2) are always true.

If $p \notin L$, eq.(1) or eq.(2) is false with over pr. 1/2

Check on $a_1, \ldots, a_k$:

If eqs (1) and (2) are true for all a_i,

then *p* is prime greater than with pr. $(1 - 1/2^k)$

BPP (III)

$\Pr[M(x) = 1] \geq 2/3$ if $x \in L$

$\Pr[M(x) = 0] \geq 2/3$ if $x \notin L$

$\Updownarrow$

$\Pr[M(x) = 1] \geq 1/2 + |x|^{-c}$ if $x \in L$

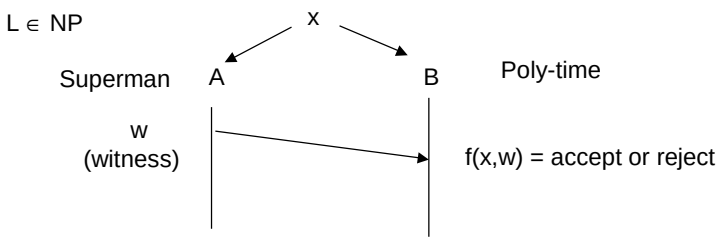
$\Pr[M(x) = 0] \geq 1/2 + |x|^{-c}$ if $x \notin L$

$\Updownarrow$

$\Pr[M(x) = 1] \geq 1 - 2^{-|x|}$ if $x \in L$

$\Pr[M(x) = 0] \geq 1 - 2^{-|x|}$ if $x \notin L$

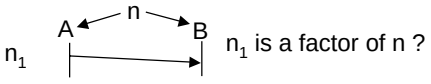
Computation & Proof (III)



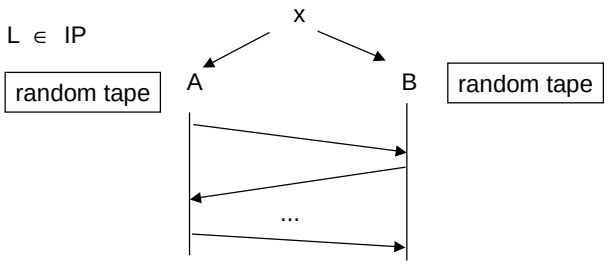
Completeness : if $x \in L$, $f(x, \exists w)$ = accept

Soundness : if $x \notin L$, $f(x, \forall w)$ = reject

(예) $L = \{ n \mid n = \text{composite} \}$, $n = n_1 n_2$

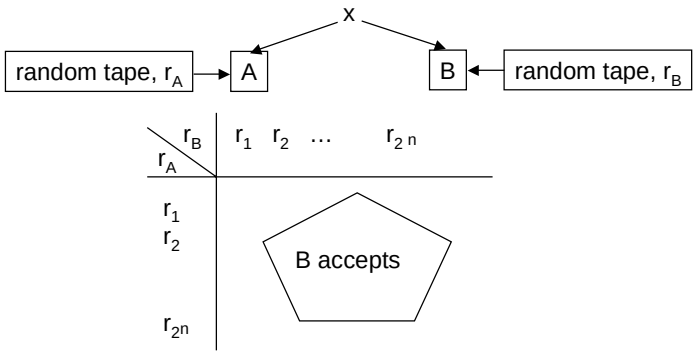


Computation & Proof (IV)



Completeness if $x \in L$, prob[B accepts x] $\geq 1 - \varepsilon$
Soundness if $x \notin L$, prob[B rejects x for $^{\forall}$A] $\geq 1 - \varepsilon$

Meaning of Probability (IP)



Prob(B accepts) ≡ Area that B accept / Total area

$\begin{cases} 1 - \varepsilon & \text{if } x \in L \\ < \varepsilon & \text{if } x \notin L \text{ for } {}^{\forall} A \end{cases}$

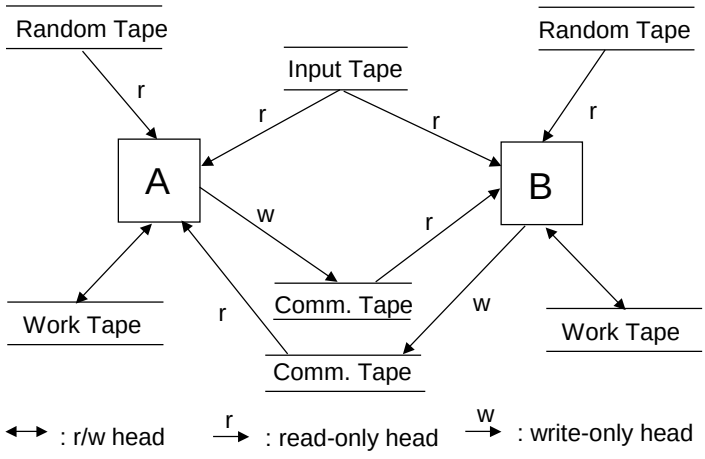
IPS

- Protocol : a pair of algorithm (A,B)
- Interactive Proof System : Protocol (A,B) satisfying completeness and soundness
- If L $\in$ IP (Interactive Poly-time), L has an IPS (Interactive Proof System).

ZKIP

- GMR(Goldwasser, Micali, Rackoff) ; Proposed at first in 1985
- ZKIP (Zero Knowledge Interactive Proof) : Between P and V,
  - Completeness : Only true P can prove V.
  - Soundness : False P' can't prove V.
  - 0-Knowledge : No knowledge transfer to V.

Turing Machine Model



Concepts of ZKIP

By Quisquater and Guillou

P knows the secret, but he doesn't want to reveal his secret.

1. V stands at point A.

2. P walks all the way into the cave, either C or D.

3. After P disappeared into the cave, V walks to point B.

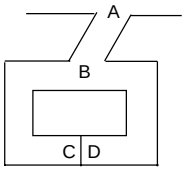
4. V shouts to P asking him either to:

(a) come out of the left passage or (b) come out of the right passage

5. P complies, using the magic words to open secret door if he has to.

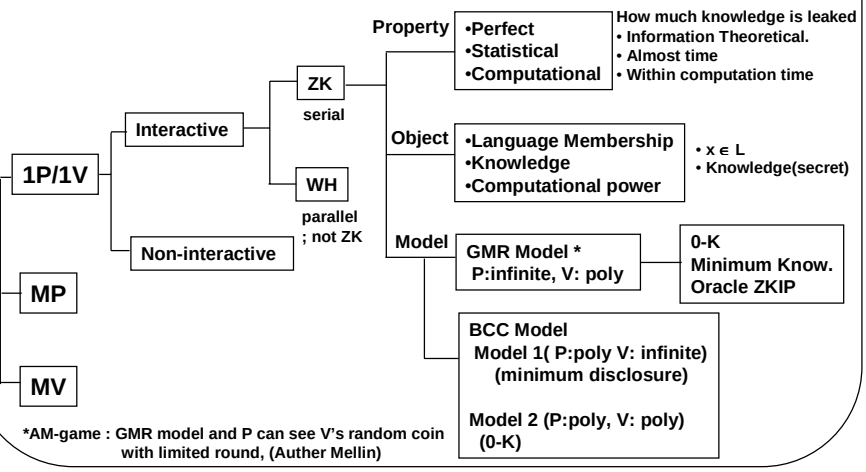
6. P and V repeat step (1) -(5) *t* times

*** P knows the magic words (secret) to open the secret door between C and D.**



0-knowledge cave

Classification of ZKPS



Indistinguishability (I)

- Family of r.v., U ={U(x)} where *x* is from L, a particular set of {0,1}*, all r.v. are taken from {0,1}* , U and V are r.v.
- Verdict who can tell a bit from U or V is limited to
  - infinite time and space : perfect
  - infinite time and polysize space : statistical
  - polysize time and space : computational

Indistinguishability (II)

- **L : Language**
- **{U(x)}, {V(x)} : family of random variable**
- **(Perfect) If for all $x \in L$, U(x) = V(x) (where "= " means "equal as random variables") , {U(x)} and {V(x)} are perfectly indistinguishable for L.**
- **(Statistical) If $\Sigma_{\alpha \in \{0,1\}^*} |Pr[U(x)=\alpha] - Pr[V(x)= \alpha]| < \varepsilon (|x|)$, {U(x)} and {V(x)} are statistically indistinguishable for L.**
- **(Computational) For all circuit C (distinguisher) with polynomial size of |x| , if $|Pr[C(U(x))=1] - Pr[C(V(x))= 1]| < \varepsilon$, {U(x)} and {V(x)} are computational indistinguishable for L.**

Way of proofing

There are many ways to prove the truth of a proposition like "I know the modular square root of V" (or any other PSPACE problem):

1. **To give the proof (i.e., to tell the square root to the verifier)**
2. **Zero-knowledge proof : to convince the verifier that the claim holds without giving him any information on the proof (and thus he cannot compute the square root).**

ZKIPs are used in identification scheme, in which a user (called the prover) proves to the verifier that he knows a certain secret, without revealing the secret, or any information on the secret.

F-S Identification(I)

- **(Preparation)**

(1) Unlike in RSA, a trusted center can generate a universal *n*, used by everyone as long as none knows the factorization.

(2) P has an RSA modulo *n=pq* whose factorization is secret.

(3) secret key : P chooses random value *S*, s.t. gcd(*S,n*)=1.(1 < *S* < *n*)

public key : P computes $I=S^2$ mod *n*, and publishes (*I,n*) as public

F-S Identification(II)

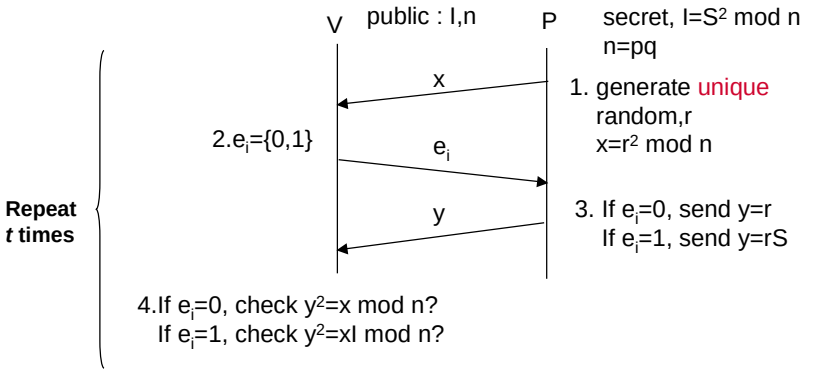
(Goal)

P has to convince V that he knows secret key *S* corresponding public key (*I,n*) (i.e., to prove that he knows a modular square root of *I* mod *n*), without revealing *S*.

F-S Identification(III)

1. P chooses random value r (1<r<n) and computes $x=r^2 \bmod n$. then sends x to V.
2. V requests from P one of the following request at random
   (a) r or (b) rS mod n
3. P sends the requested information to V.
4. V verifies that he received the right answer by checking whether
   (a) $r^2 = x \bmod n$ or (b) $(rS)^2 = xI \bmod n$
5. If verification fails, V concludes that P does not know S, and thus he is not the claimed party.
6. This protocol is repeated t (usually 20 or 30) times, and if in all of them the verification succeeds, V concludes that P is the claimed party.

F-S Identification(IV)



* commitment-witness-challenge-response-verification and repeat

Security of F-S scheme

(1) It is assumed that computing S is difficult, actually the difficulty is equivalent to that of factoring n.

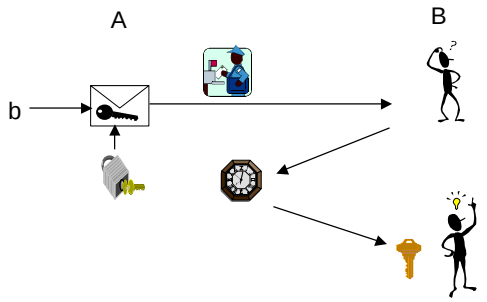
(2) Since P doesn't know in advance (when he chooses r or rS mod n) which question V will ask, he can't choose the required choice. He can succeed in guessing V's question with prob. 1/2 for each question, and thus V can catch him in half of the times, and fails to catch him in half of the times. The protocol is repeated t times,and thus the prob. that V fails to catch P in all the times is only 2^{-t}, which is exponentially reducing with t. (t=20 or 30)

F-S scheme is ZKIP

- **The F-S protocol convinces V that P knows the square root of *I*, without revealing any information on *S*. However, V gets <u>one bit</u> of information : <u>he learns that *I* is a quadratic residue</u>**

Bit Commitment(I)

- Basic component of many cryptographic protocols
  - Commit stage : A commits B to a bit b, that B has no idea what b is.
  - Revealing stage : B can verify that committed bit is from A.



Bit Commitment(II)

Def) S,V : probabilistic poly time TM

- Commit Phase : S selects b $\in_U$ {0, 1} and sends it to V.

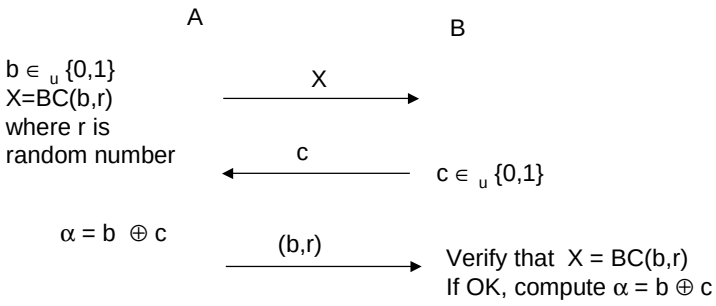
- Reveal Phase : S reveals b to V and V finally accept or rejects.

(1) At commit phase, an adversary A tries to compute b like V, probability to derive *b* is negligible small.

(2) After A did commit phase like S, then revealing b=0 or b=1 at the reveal phase is negligible small even if he has an unlimited power.

(Theorem) We can construct BC for a given 1 –way ft.

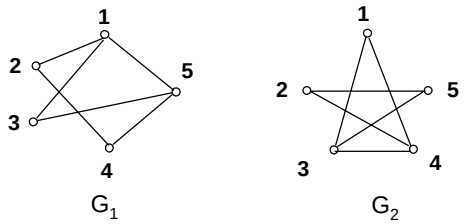
Coin flipping by BC



α: coin flipping result
Each side can't change the value of α at favour.

GI(Graph Isomorphism)

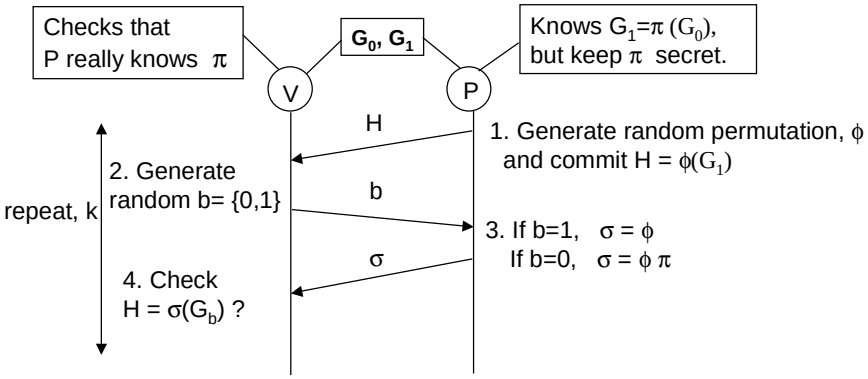
- **(Def) G={V,E}=((1,…,n),({(i,j)})), n vertex**
- **∃ a 1-1 and onto mapping ϕ keeping the incidence relation of Graph G_1 and G_2.**



ϕ =(1, 2, 3, 4, 5, → 4, 2, 1, 5, 3) $G_2 = \phi(G_1)$

GI belongs to NP (Non deterministic Polynomial).

ZKIP using GI (I)



Random Self-reducibility :
average = worst complexity
(*e.g*) GI,DL,QRA

ZKIP using GI(II)

- **(Completeness) : If G_0 and G_1 are isomorphism, there exists π and V accepts P with prob. 1.**
- **(Soundness) : If G_0 and G_1 are not isomorphism, H is not isomorphic to G_0 nor G_1 at step 1. Thus, V selects b at random, the prob. of passing validation step 4 is 1/2. If repeats k times. Prob. of acceptance is $1/2^k$ ($<\varepsilon |x|$).**
- **(0-Kness) : Done by Simulator**

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